A Theory of the Size and Investment Duration of Venture Capital Funds

Dawei Fang[†]

Centre for Finance, Gothenburg University

Abstract: We take a portfolio approach, based on simple agency conflicts between a general partner (GP) and limited partners (LPs), to analyze the size and investment duration of a venture capital fund. A large fund, with investment duration restrictions and limited GP's profit sharing, minimizes agency conflicts but may force the GP to offer LPs positive rents, even if the GP has all the bargaining power. When rents are sufficiently large, the GP trades off efficiency for reduced rents by establishing a small fund if projects have moderate payoff potential or by establishing a large fund with no investment duration restrictions if projects have high payoff potential.

Keywords: Agency problem, Limited partnership, Mechanism design, Portfolio financing, Sequential timing of investments, Venture capital

JEL Classifications: G24, D82, D86

[†] Contact email: dawei.fang@cff.gu.se.

1 Introduction

Venture capital (VC) funds in the U.S. are predominantly set up as limited partnerships (see Gompers and Lerner 1999, and Cumming and Johan 2009). Limited partnerships have a finite lifespan, which is mandated by law in the U.S., and the standard lifespan of a U.S. VC fund is 10-13 years (see Sahlman 1990, Phalippou 2007, and Cumming and Johan 2009). The life of a VC fund can be broadly partitioned into an "investment stage" and a "divestment stage." During the investment stage, a venture capitalist, who serves as the general partner (GP) in the limited partnership, searches for and invests in projects. During the divestment stage, after investments have been brought to fruition, the GP sells the investments of the VC fund in order to realize capital gains.

In the past, the partition of these stages was rather informal: the limited lifespan only imposes an upper bound on the *total* time taken to invest and divest, without further specifying the duration of each stage. However, recently many VC funds have begun to contractually restrict the duration of the investment stage, e.g., the GP cannot make new investments after 5 or 6 years from the start of the partnership. Such an arrangement is considered by Kandel, Leschchinskii, and Yuklea (2011) as evidence of the severity of the potential agency problem in the investment stage. One agency problem that receives recent empirical attention is the GP's moral hazard of increasing risk taking after early investment failures. As shown by Dass, Hsu, Nanda, and Wang (2012), GPs, especially young GPs, invest in more projects after early investment failures and incur more failures in the late investments, and a VC fund's percentage of successful exists, through either IPO or merger and acquisitions, is much lower in the fund's later investments than earlier investments.

In this paper, we take a portfolio approach to analyze the GP's *ex post* investment strategy (moral hazard) and investigate the GP's *ex ante* choice of the size and investment duration of a VC fund. More specifically, we address the following questions: Would the restrictions imposed on a VC fund's investment duration help discipline the GP's *ex post* investment behavior? If so, why such restrictions are not imposed by all VC funds? What are the benefits and costs of using such restrictions? Does the fund size affect the resolution of the agency problem? What are the benefits and costs of having a smaller versus a larger VC fund? How do project profitability, investment focus, and the bargaining power held by limited partners (LPs) against the GP affect the VC fund's size and investment duration?[1] What affects LPs' payoffs?

In our model, because of an *ex ante* adverse selection problem, the GP can only raise external capital by using a contract that offers him no payoff unless he can create a profit for the VC fund. This option-like characteristic of GP compensation gives the GP an inclination towards risk-shifting. As all *ex post* inefficiency is borne by the GP *ex ante*, to reduce financing costs, the GP has an incentive to discipline his *ex post* investment behavior by *ex ante* contracting. We show that, to minimize agency costs, the *ex ante* financing arrangement has to satisfy four conditions: (i) financing all projects by joint liability, (ii) raising enough funds for all projects, (iii) using investment duration restrictions, and (iv) limiting the GP's profit sharing. Although this financing arrangement is implementable, it may force the GP to give LPs positive rents. If the rents offered to LPs are too large, the GP may turn to less efficient but rent-saving financing arrangements by, for example, deliberately limiting the amount of capital raised or imposing no restrictions on investment duration.

Our analysis shows that, without restrictions imposed on a VC fund's investment duration, the GP, with enough capital to invest in multiple projects, has an incentive to play *strategic sequential timing of investments* when the projects he has access to happen to be all risky but negative-NPV investments (bad projects). Particularly, he has an incentive to invest in some bad projects first and gamble on the other bad projects upon foreseeing the failure of his earlier investments. Accordingly, there exists a positive probability that all bad projects are undertaken. To contract off the strategic sequential timing of investments, the VC fund has to contractually shorten the investment duration so that the GP has to make his investment decisions on all projects simultaneously. However, investment duration restrictions *per se* can only contract off the GP's strategic sequential timing behavior but cannot guarantee an improvement in efficiency. If the GP has too much profit sharing, with investment duration restrictions imposed and enough funds at the GP's hand, the overinvestment problem can even be exacerbated; in that case, the GP has an incentive to invest in all bad projects at the first place. Therefore, it is important to limit the GP's profit sharing when investment duration restrictions are imposed. However, when each project's payoff potential, defined as the rate of return of a project if successful, is high, such an arrangement

[1] We use "investors" and "LPs" interchangeably in this paper.

may force the GP to offer LPs rents, even though LPs have no bargaining power. Hence, the GP, in essence, has to trade off between saving agency costs and saving rents by choosing between more restrictive GP compensation with investment duration restrictions and less restrictive GP compensation with no investment duration restrictions. It turns out that the GP chooses the latter if projects' payoff potential is sufficiently high.

A smaller fund size may be chosen by the GP only if minimizing agency costs ends in too much rents to LPs. The advantage of a smaller fund, relative to a larger fund with no investment duration restrictions, comes from its ability to reduce overinvestment: When all projects happen to be bad, the GP can only invest in part of them with a smaller fund. However, a smaller fund also has its disadvantage in terms of causing underinvestment: When the GP has access to a large number of positive-NPV projects (good projects), he can only invest in part of them. A smaller fund's advantage outweighs its disadvantage only if projects' payoff potential is not too high. Combined with the earlier results, our model thereby implies that the GP is more likely to establish a larger VC fund with investment duration restrictions if projects have low payoff potential, establish a smaller VC fund if projects have moderate payoff potential, and establish a larger VC fund with no investment duration restrictions if projects have high payoff potential.

Our model also shows that an increase in investment focus, measured by an increase in project quality correlation, tends to induce the GP to establish a larger fund. This is mainly because an increase in project quality correlation increases the probability that all projects are good, which exacerbates the underinvestment problem associated with a smaller fund.

Our analysis also establishes that an increase in LPs' bargaining power tends to induce the GP to use the more efficient financing arrangement, i.e., a larger VC fund with investment duration restrictions. This is because giving LPs some bargaining power enables LPs to ask for positive rents. In this case, the disadvantage of the more efficient financing arrangement, in terms of rent-offering, becomes less of a problem for the GP, since the GP now has to offer LPs positive rents anyway. This result implies that increasing LPs' bargaining power increases the investment efficiency of VC funds.

Our work contributes to two strands of the literature. First, our work contributes to the theoretical literature that analyzes the effect of pooling on investment incentives and optimal contracting. Diamond (1984) shows that by changing the cash flow distribution, investment pooling makes it possible to design contracts that incentivize the agent to monitor investments properly. Bolton and Scharfstein (1990) and Laux (2001) show that tying investment decisions together can create "inside wealth" for the agent undertaking investments, which reduces the limited liability constraint and helps design more efficient contracts. Tirole (2006, Chapter 4.2) points out that borrowing capacity can be boosted by cross-pledging when the entrepreneur has the moral hazard of shirking. Our work, however, endogenizes the timing of investments and considers the moral hazard problem associated with the agent's strategic sequential timing of investments when he manages a portfolio of projects, a missing piece from the existing literature on investment pooling.

Second, this paper contributes to an emerging literature analyzing private equity and venture capital fund structures. Kandel, Leschchinskii, and Yuklea (2011) argue that the finite lifespan of a VC partnership can induce a GP to make inefficient investments in risky projects. However, their arguments are based on exogenously specified GP compensation structure and VC fund duration, which have both been endogenized in this paper. Inderst, Mueller, and Münnich (2007) argue that pooling private equity investments together in a fund helps a GP commit to efficient liquidation decisions, in a way similar to the winner-picking model of Stein (1997). Their mechanism relies on making the fund capital-constrained, which is not always optimal in our model. Axelson, Strömberg, and Weisbach (2009) study the GP's financing of a portfolio of projects. They show that in the pure *ex ante* financing setting, it is never optimal to make the GP capital-constrained, and they find that the optimal financing structure is a combination of *ex ante* pooled financing and *ex post* deal-by-deal financing. In contrast, this paper shows that in the pure *ex ante* financing setting, although pooled financing can increase efficiency, the GP may prefer having the fund capital-constrained in order to save rents. Fulghieri and Sevilir (2009) study the optimal size and scope of a venture capitalist's portfolio by focusing on the double-sided moral hazard problem between the venture capitalist and entrepreneurs. They show that the venture capitalist finds it optimal to limit portfolio size when projects have high payoff potential and the venture capitalist expands a portfolio size only when project fundamentals are more moderate and when he can form a sufficiently focused portfolio. In contrast, this paper abstracts from the effort incentive problem and focuses on the GP-LP relationship rather than the GP-entrepreneur relationship. Our work shows that, even with exogenously fixed project quality, heterogeneous fund sizes can be an equilibrium phenomenon and the venture capitalist deliberately limits the fund size only when projects have *moderate* (rather than high) payoff potential.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 analyzes the model and determines the optimal size and investment duration of a VC fund. Section 4 presents the extensions of the model. Section 5 provides the empirical implications of the model. Section 6 concludes. All the proofs are relegated to the Appendix.

2 The Model

There are three types of players in the model: a general partner (GP), investors who are perfectly competitive, and an infinite supply of fly-by-night operators. All players are risk-neutral, with no discounting of the future, and have access to a storage technology yielding the risk-free interest rate equal to zero.

At $t = 1$, two projects arrive. Each requires an investment of I to be undertaken. Each project can be either good (G) or bad (B). The quality of each project is only observable to the GP at $t = 1$, and for the time being, we assume that the two projects have independent quality. We relax this independence assumption in model extensions. Investors only know that with probability λ, a project is good, and with probability $1 - \lambda$, a project is bad. A good project, if undertaken, produces cash flow $R > 0$ (succeed) with certainty, while a bad project, if undertaken, produces cash flow 0 (fail) with probability $1 - p$ and cash flow R (succeed) with probability p, where

$$R > I > pR. \tag{1}$$

Condition (1) implies that good projects are positive NPV investments while bad projects are negative NPV investments. For the time being, we further assume that

$$R > 2I, \tag{2}$$

so the costs of investing in both projects can be covered when at least one investment succeeds. We relax this assumption in model extensions. Note that conditions (1) and (2) further imply that

$$p < \frac{1}{2}. \tag{3}$$

We assume that a project has invariant quality no matter whether it is undertaken at $t = 1$ or $t = 2$. Notably, this assumption is stronger than what we require for our results; all our results are preserved if we instead assume that a good project turns bad if it is undertaken at $t = 2$. In addition, so long as the decline in a bad project's probability of success is not big if it is undertaken later, there would be no change in the qualitative nature of our results. We impose the stronger "invariant quality" assumption simply for the purpose of showing that the empirically documented lower percentage of successful exits generated by a VC fund's later investments can be due to the GP's moral hazard of strategic sequential timing of investments but not due to a decline in candidate projects' quality.

We assume that a project's cash flow is realized two periods after it is undertaken, so if the GP invests I in a project at $t = 1$ ($t = 2$), this project's cash flow is realized at $t = 3$ ($t = 4$). This assumption makes it impossible for the GP to (self-)finance later ($t = 2$) investment by the proceeds from earlier ($t = 1$) investment.

Although the GP obtains no proceeds from any earlier investments at $t = 2$, we assume that at the start of $t = 2$, the GP obtains information on the progress of earlier investments. For analytical convenience, we assume that at the start of $t = 2$, the GP knows whether his investments made at $t = 1$ would eventually succeed or fail.[2]

We assume that the GP has no money of his own.[3] We focus on the *ex post* information asymmetry regarding project quality by assuming that the GP has to finance his projects at $t = 0$, i.e., before the arrival of projects.[4] Thus, *ex ante* the GP has no information advantage over investors on project quality. To raise funds K at $t = 0$, where K is endogenous, the GP issues a security $s_I(x)$ backed by the final cash flow x from the investments and keeps the residual security $s_{GP}(x) = x - s_I(x)$. Investors trade K for s_I if they can at least break

[2]This assumption is consistent with the point made by Dass, Hsu, Nanda and Wang (2012): "The final outcome of the early investments may not be known to either the investors or even the venture capitalists themselves when the venture capitalists decide to invest in new projects. However, it is likely that VC fund managers have information on the likelihood of the eventual outcome of an investment project after investing in the project."

[3]Such an assumption is also adopted in Axelson, Strömberg, and Weisbach (2009). Although many VC funds have capital contribution provisions that require the GP to contribute 1% of the committed capital, the GP is frequently permitted to borrow money from the VC fund to meet his capital contribution requirements (see Harris 2010).

[4]This assumption can be justified on the grounds that VC fundraising is time consuming.

even. Admissible securities have to satisfy the following limited liability and monotonicity conditions:

Limited Liability (LL): $0 \leq s_I(x), s_{GP}(x) \leq x$ for all $x \geq 0$,

Monotonicity (M): $s_I(x)$ and $s_{GP}(x)$ are nondecreasing in x.[5]

In practice, it is common that the GP earns no performance-based compensation before LPs are paid in full (see Sahlman 1990). This option-like characteristic of GP compensation leads to the GP's risk-shifting tendency that is critical for our results. In the same way as Axelson, Strömberg, and Weisbach (2009), we impose this option-like characteristic on GP compensation by an adverse selection problem: at $t = 0$, without contracts as a signalling device, investors cannot distinguish capable GPs from unskilled fly-by-night operators who cannot bring success to any project but can store money at the riskless interest rate. Facing this problem, investors are not willing to accept any contract that offers the GP a strictly positive payoff by storing money at the riskless interest rate, since if they were, fly-by-night operators would have incentives to offer investors such a contract and as the supply of fly-by-night operators is, by assumption, infinitely large, the probability that investors meet a fly-by-night operator would be one, in which case investors must lose money. Therefore, any contract that can possibly be accepted by investors must satisfy the following condition.

No-Profit-No-GP-Earning (NP): for invested capital K, $s_{GP}(x) = 0$ whenever $x \leq K$.

Condition (NP) ensures that fly-by-night operators earn no profit from fundraising, so a capable GP is separated from fly-by-night operators by using a contract that satisfies (NP).

Finally, we assume that covenants and provisions can be used in the limited partnership agreement to impose investment duration restrictions. If such restrictions are imposed, projects can only be undertaken at $t = 1$; if otherwise, projects can be undertaken either at $t = 1$ or at $t = 2$.[6] This assumption is consistent with the VC practice. We will talk about how investment duration restrictions are implemented in practice in Section 5.

[5] These assumptions are quite common in the financial literature on security design, e.g., in Harris and Raviv (1989) and Nachman and Noe (1994). (M) guarantees that the GP cannot increase his own payoff by manipulating the ex post cash flows, either through borrowing money from a third party or burning money.

[6] Alternatively, to restrict investment duration, the limited partnership can specify a short lifespan, e.g., the partnership must be dissolved at $t = 3$ with all unfinished projects transferred to the LPs. In this case, knowing that investments made at $t = 2$ cannot be successfully exited at $t = 3$, the GP has no incentive to make new investments at $t = 2$. However, as in practice, there is little heterogeneity in the lifespan of VC funds within a country, we have not followed this interpretation in the main text.

3 Model Analysis

3.1 The second-best financing arrangement

3.1.1 Implementation

At $t = 1$, there are three states of the world, distinguished by the combination of two projects' quality: (1) both projects are good – state GG, (2) one project is good while the other one is bad – state GB,[7] and (3) both projects are bad – state BB. It is noticeable that, due to condition (NP), the first-best world, in which the GP invests in all good projects and no bad projects in all states, is unattainable: in state BB, the GP always has an incentive to invest in at least one bad project, since otherwise he earns nothing. Thus, efficiency loss caused by undertaking one bad project in state BB is inevitable. It is thus straightforward that the second best is attained if and only if the GP has incentives to

(i) invest in both projects in state GG;

(ii) invest in only the good project in state GB;

(iii) invest in only one bad project in state BB.

We call the financing arrangement that induces the GP to follow the above prescribed investment behavior *the second-best arrangement*.

It is important to note that, to induce the GP to follow the above prescribed investment behavior, the ex-ante financing arrangement must satisfy the following two conditions:

(a) sufficient funds raised for investing in two projects (larger fund size);

(b) putting the two projects together under one roof (cross-pledging).

The first condition – larger fund size – is necessary to allow the GP to invest in both projects in state GG; it requires the GP to raise at least $2I$ for the fund. The second condition – cross-pledging – is necessary to induce the GP to abandon a bad project, since if the GP instead uses stand-alone financing for each project, given that each contract has to satisfy condition (NP), the GP will never abandon any bad project.

[7] Note that given that the two projects are identical, it is irrelevant which of the two projects is good. We therefore treat these two separate but symmetric cases effectively as a single case.

In addition, imposing the following investment duration restrictions is beneficial for the implementation of the second best:

(c) no new investments allowed at $t = 2$ (investment duration restrictions).

The reason why investment duration restrictions benefit the implementation of the second best is because such restrictions contract off the GP's strategic sequential timing of investments and thereby make it easier to design the incentive compatible contract. Below we explain the reason in detail. One the one hand, without investment duration restrictions, if the GP has raised sufficient funds for the investment of two projects, then in state BB, he has an incentive to invest in one bad project at $t = 1$ first and if he foresees the failure of this investment at the start of $t = 2$, he has an incentive to invest in the other bad project at $t = 2$ unless he earns nothing from the success of the second bad project. The latter requires $s_{GP}(R) = 0$. To induce the GP to abandon the bad project in state GB, his earning from investing in only the good project, which is $s_{GP}(R+I)$, must be weakly greater than his earning from investing in both the good and the bad project, which is $ps_{GP}(2R)+(1-p)s_{GP}(R)$. When $s_{GP}(R) = 0$, the latter becomes $ps_{GP}(2R)$. Therefore, to implement the second best without investment duration restrictions, the GP security has to satisfy the following GP's incentive compatibility constraints.[8]

(IC^{Δ}_{GB}): $s_{GP}(R + I) \geq ps_{GP}(2R)$

(IC^{Δ}_{BB}): $s_{GP}(R) = 0$

On the other hand, with investment duration restrictions, the GP cannot play strategic sequential timing of investments. In that case, to implement the second best, the GP security only has to satisfy the following GP's incentive compatibility constraints.

(IC_{GB}): $s_{GP}(R + I) \geq ps_{GP}(2R) + (1 - p)s_{GP}(R)$

(IC_{BB}): $ps_{GP}(R + I) \geq p^2 s_{GP}(2R) + 2p(1 - p)s_{GP}(R)$

The left-hand side of (IC_{GB}) is the GP's expected payoff in state GB if he only invests in the good project. The right-hand side of (IC_{GB}) is the GP's expected payoff in state GB if he invests in both projects. When (IC_{GB}) holds, the GP only invests in the good project in state GB. The left-hand side of (IC_{BB}) is the GP's expected payoff in state BB if he

[8]As we assume that good projects will succeed for sure, the incentive compatibility constraints that induce the GP to invest in good projects are implied by the monotonicity of the GP security $s_{GP}(\cdot)$, which has been imposed already.

only invests in one bad project. The right-hand side of (IC_{BB}) is the GP's expected payoff in state BB if he invests in both projects. When (IC_{BB}) holds, the GP only invests in one bad project in state BB with investment duration restrictions.

Note that, if s_{GP} satisfies both (IC^{Δ}_{GB}) and (IC^{Δ}_{BB}), it must satisfy (IC_{GB}) and (IC_{BB}) as well, but not vice versa. As by assumption, $R > 2I$, condition (NP) only requires $s_{GP}(2I) = 0$ but does not require $s_{GP}(R)$ to be zero. Therefore, by restricting investment duration, the security design problem for the second-best implementation is relaxed and we show later on that it is (weakly) more costly for the GP, in terms of rent-offering, to implement the second best without investment duration restrictions than with such restrictions.

For the ease of exposition, we call a capital-unconstrained cross-pledging financing arrangement with investment duration restrictions *a short duration arrangement*. The above discussion shows that a short duration arrangement with the GP security satisfying (IC_{GB}), (IC_{BB}), and other conditions described previously, i.e., (LL), (M), and (NP), is the best way of implementing the second best. Below we derive the second-best contract. Note that, having a financial slack does not offer the GP any additional benefits, so without loss of generality, we assume that the GP does not raise more than what he would possibly use.[9] Thus, here, to possibly invest in two projects, the GP raises $2I$ but not more than that.

With cross-pledging, the final cash flow x can potentially take six different values, i.e., $x \in \{0, I, 2I, R, I + R, 2R\}$. The GP security, s_{GP}, must specify the GP's payoff for each of these six cash flow realizations, among which $s_{GP}(0) = s_{GP}(I) = s_{GP}(2I) = 0$ according to condition (NP). It is noticeable that (IC_{BB}) implies (IC_{GB}), so to derive the second-best contract, we can drop (IC_{GB}) from the program. Therefore, inherited from the previous analysis, the second-best financing arrangement must be a short duration arrangement with the GP security, s_{GP}, solving the following program:

$$\max_{s_{GP}(x)} E[s_{GP}(x)] = \lambda^2 s_{GP}(2R) + \left[2\lambda(1-\lambda) + (1-\lambda)^2 p\right] s_{GP}(R+I), \tag{4}$$

[9] In a setting of dynamic contracting with renegotiation, in equilibrium the agent may have a financial slack to deal with the cash flow risk, e.g., in DeMarzo and Fishman (2007). However, in our model, although investments can be sequential and cash flows can be risky, the cash flow risk only affects the financial status of the fund *after* all the investment decisions have been made, so the potential benefit of having a financial slack disappears in our model.

subject to (LL), (M), (NP), (IC_{BB}), and the following investors' break-even condition

$$E[x - s_{GP}(x)] \geq I \qquad (BE)$$

There are two possible payoffs to the GP in the maximand. The first payoff, $s_{GP}(2R)$, occurs only in state GG. The second payoff, $s_{GP}(R+I)$, occurs both in state GB and in state BB with the success of the undertaken bad project.[10]

Our first proposition shows that the GP obtains all the investment surplus (the second-best surplus) by using the second-best arrangement if and only if projects' payoff potential, represented by R/I, is not too high; if it is sufficiently high, the GP has to offer positive rents to LPs (although LPs are perfectly competitive). The security design is relegated to the Appendix.

PROPOSITION 1 *There are critical values* $\{\pi_0(p,\lambda), \pi_{rent}(p,\lambda)\}$ *(defined in the Appendix), with* $\pi_0(p,\lambda) < \pi_{rent}(p,\lambda)$*, such that*

(i) for projects with lower payoff potential (that is, $R/I < \pi_0(p,\lambda)$*), the second-best arrangement is infeasible;*

(ii) for projects with mildly low payoff potential (that is, $\pi_0(p,\lambda) \leq R/I \leq \pi_{rent}(p,\lambda)$*), the second-best arrangement is feasible and by using the second-best arrangement, the GP leaves no rents to LPs and obtains all the second-best surplus, expressed as follows:*

$$[2\lambda + (1-\lambda)^2 p]R - (1+\lambda^2)I; \tag{5}$$

(iii) for projects with higher payoff potential (that is, $R/I > \pi_{rent}(p,\lambda)$*), the second-best arrangement is feasible and by using the second-best arrangement, the GP has to leave LPs with positive rents and thereby obtains an expected payoff strictly less than the second-best surplus. (The GP's expected payoff in this case is enclosed in the Appendix.)*

3.1.2 LPs' Rents

As been discussed before, the first best is unattainable and thus the second-best surplus is the maximal expected payoff the GP could possibly obtain. However, Proposition 1 shows

[10] If in state BB, the undertaken bad project fails, the final cash flow is I. By condition (NP), $s_{GP}(I)$ equals zero and is hence missing in the maximand.

that the second-best arrangement may force the GP to give LPs positive rents, which may drive the GP's expected payoff below the second-best surplus.

To see why the second-best financing arrangement may give LPs positive rents, note that (IC_{BB}) is equivalent to

$$s_{GP}(R+I) \geq ps_{GP}(2R) + 2(1-p)s_{GP}(R). \tag{6}$$

The left-hand side of (6) is less than $s_{GP}(R) + I$ because of condition (M), so the following condition is implied by (IC_{BB}) and (M):

$$s_{GP}(R) + I \geq ps_{GP}(2R) + 2(1-p)s_{GP}(R), \tag{7}$$

which can be rearranged into

$$I \geq ps_{GP}(2R) + (1-2p)s_{GP}(R). \tag{8}$$

As $0 < p < 1/2$, both p and $(1-2p)$ are positive. Thus, (8) implies that, in order to satisfy conditions (IC_{BB}) and (M), $s_{GP}(2R)$ and $s_{GP}(R)$ cannot be both set too high relative to I. This restricts the GP's sharing of the marginal cash flows in the region $[I, R]$ as well as the GP's upside sharing and thus creates a lower bound on the return pledged to LPs. When R/I is sufficiently large, this lower bound may well exceed I.

As the GP may not earn the second-best surplus by using the second-best arrangement, it is so far unclear whether the second-best arrangement offers the GP the highest expected payoff compared to less efficient but possibly rent-saving financing arrangements and it thus remains the question whether the second-best arrangement would be the equilibrium financing arrangement.[11] We tackle these questions in the rest of this section, beginning with the examination of the third-best financing arrangement, the GP's best alternative choice in terms of efficiency.

3.2 The third-best financing arrangement

3.2.1 Alternative financing arrangements

Recall from the discussion in Section 3.1 that there are four conditions for the second-best arrangement: (i) larger fund size, (ii) cross-pledging, (iii) investment duration restrictions,

[11] By Proposition 1, the GP enjoys the second-best surplus when projects' payoff potential is mildly low, so we know for sure that in this case the second-best arrangement is the equilibrium financing arrangement.

and (iv) the incentive compatibility constraint (IC_{BB}).[12] By relaxing one or more conditions, we can eventually obtain four mutually exclusive alternative financing arrangements, listed as follows:

(1) *the capital-constrained arrangement*, under which the GP raises funds sufficient for investing in *only* one project;

(2) *the long duration arrangement*, under which the GP finances the two projects by cross-pledging and the VC fund does not have investment duration restrictions;

(3) *the stand-alone arrangement*, under which the GP does not use cross-pledging but finances the two projects by using two separate contracts;

(4) *the non-second-best short duration arrangement*, which is everything the same as the second-best arrangement except that the GP security violates (IC_{BB}).

Among the above four alternative financing arrangements, the one that leads to the largest investment surplus is the third-best arrangement. In what follows, we derive the third-best arrangement by comparing the investment surplus under each of these four financing arrangements. During the process, we also keep an eye on LPs' rents.

A. The capital-constrained arrangement

By using the capital-constrained arrangement, the GP can at most undertake one project. Because of condition (NP), the GP can only obtain a positive payoff if his undertaken project is successful. Thus, the GP always has an incentive to undertake one project in any state. In addition, in state GB, as the good project has a higher probability of success than the bad one, the GP always picks up the good project. Therefore, the GP's investment behavior by using the capital-constrained arrangement is as follows: (i) in state GG, undertake one good project, (ii) in state GB, undertake one good project, and (iii) in state BB, undertake one bad project. The next proposition shows that the capital-constrained arrangement is feasible so long as projects' payoff potential is not too low, and by using a simple contract, the GP gives LPs no rents under the capital-constrained arrangement.

PROPOSITION 2 *There exists a critical value $\pi_C(p, \lambda)$ (defined in the Appendix) such that*

[12] The other incentive compatibility constraint (IC_{GB}) is implied by (IC_{BB}).

(i) for projects with lower payoff potential (that is, $R/I < \pi_C(p,\lambda)$), the capital-constrained arrangement is infeasible;

(ii) for projects with higher payoff potential (that is, $R/I \geq \pi_C(p,\lambda)$), the capital-constrained arrangement is feasible and the optimal GP security s_{GP} (which is not always unique) is given by

$$s_{GP}(x) = \begin{cases} 0 & \text{if } x \leq I \\ \alpha_C(x-I) & \text{if } x > I, \end{cases} \tag{9}$$

where $0 \leq \alpha_C \leq 1$ and α_C is determined by making the investors' break-even condition (BE) bind. By using this security under the capital-constrained arrangement, the GP gives LPs no rents and the GP's expected payoff equals

$$\left[2\lambda - \lambda^2 + (1-\lambda)^2 p\right] R - I. \tag{10}$$

The optimal security presented by Proposition 2 coincides with GP compensation in practice: the returns from investments are first used to pay back LPs' committed fund capital and afterwards, if there are any profits, the profits are split between LPs and the GP according to a predetermined ratio.

B. The long duration arrangement

By using the long duration arrangement, the GP can at most undertake two projects. As been discussed in Section 3.1.1, under the long duration arrangement, in state BB, the GP has an incentive to invest in one bad project at $t = 1$ first and invest in the other bad project upon foreseeing the failure of his earlier investment at $t = 2$ unless $s_{GP}(R) = 0$. However, the next lemma shows that a long duration arrangement with $s_{GP}(R) = 0$ is never adopted in equilibrium.

LEMMA 1 *Suppose $R > 2I$. Then the GP never uses a long duration arrangement with $s_{GP}(R) = 0$ in equilibrium.*

Lemma 1 implies that, although security design can alleviate agency conflicts due to the strategic sequential timing of investments, it is never adopted in equilibrium. This is because the strategic sequential timing of investments enabled by the long investment duration calls for stronger constraints imposed on security design to minimize agency conflicts and thereby makes it more costly, in terms of rent-offering, for the GP to minimize agency conflicts. Thus, in what follows, we focus on the long duration arrangement with $s_{GP}(R) > 0$. For expositional simplicity, in the rest of this paper, except when we talk about the extension of the model to the case where $R < 2I$ and accordingly $s_{GP}(R) = 0$ due to (NP), by saying the long duration arrangement, we mean the long duration arrangement with $s_{GP}(R) > 0$.

Efficiency maximization under the long duration arrangement requires the GP security to induce the GP to (i) invest in the two good projects in state GG, (ii) invest only in the good project in state GB, and (iii) in state BB, if the GP foresees at $t = 2$ the success of his earlier investment in one bad project, abandon the other bad project. Among the three requirements, (i) will be satisfied for sure so long as the GP security satisfies (M), while (ii) and (iii) will be satisfied if and only if the GP security satisfies the incentive compatibility constraint (IC_{GB}) presented in Section 3.1.1. The next proposition shows that so long as projects' payoff potential is not too low, there exists an optimal GP security under the long duration arrangement that satisfies (IC_{GB}) and gives the GP all the surplus from the prescribed investment behavior.

Proposition 3 *There exists a critical value $\pi_L(p, \lambda)$ (defined in the Appendix) such that*

(i) for projects with lower payoff potential (that is, $R/I < \pi_L(p, \lambda)$), the long duration arrangement (with $s_{GP}(R) > 0$) is infeasible;

(ii) for projects with higher payoff potential (that is, $R/I \geq \pi_L(p, \lambda)$), the long duration arrangement (with $s_{GP}(R) > 0$) is feasible and the optimal GP security s_{GP} (which is not always unique) is given by

$$s_{GP}(x) = \begin{cases} 0 & \text{if } x \leq 2I \\ \alpha_L(x - 2I) & \text{if } x > 2I, \end{cases} \tag{11}$$

where $0 \leq \alpha_L \leq 1$ and α_L is determined by making the investors' break-even condition (BE) bind. By using this security under the long duration arrangement, the GP gives LPs no rents and the GP's expected payoff equals

$$\left[2\lambda + (1-\lambda)^2(2-p)p\right] R - \left[2 - 2\lambda(1-\lambda) - (1-\lambda)^2 p\right] I. \tag{12}$$

Like the optimal security under the capital-constrained arrangement, the optimal security presented by Proposition 3 also fits GP compensation in practice.

C. The stand-alone arrangement and the non-second-best short duration arrangement

By using the stand-alone arrangement, the GP can at most undertake two projects. Since for each project, the contract has to satisfy (NP), the GP has an incentive to undertake both projects in any state. The corresponding investment surplus is expressed as

$$2\left[\lambda + (1-\lambda)p\right]R - 2I. \tag{13}$$

By using the non-second-best short duration arrangement, the GP undertakes two bad projects in state BB, so the investment surplus is at most equal to the first-best surplus less the efficiency loss from undertaking two bad projects in state BB; in other words, the investment surplus is bounded from above by

$$\lambda^2(2R - 2I) + 2\lambda(1-\lambda)(R - I) + (1-\lambda)^2(2pR - 2I). \tag{14}$$

Recall that the long duration arrangement only causes inefficiency in state BB, in which case the GP invests in $(2-p)$ bad projects on average. However, under both the stand-alone and the non-second-best short duration arrangement, the GP invests in two bad projects in state BB. Hence, both the stand-alone and the non-second-best short duration arrangement are less efficient than the long duration arrangement. In addition, by Proposition 3, the GP gives LPs no rents by using the long duration arrangement. Therefore, the GP must prefer the long duration arrangement to both the stand-alone and the non-second-best short duration arrangement. This can be further confirmed by noting that both (13) and (14) are smaller than (12). The next proposition thus follows.

PROPOSITION 4 *Compared to the long duration arrangement, both the stand-alone and the non-second-best short duration arrangement are less efficient and the GP never adopts them in equilibrium.*

We can now eliminate the stand-alone and the non-second-best short duration arrangement from the list of the third-best candidates. Thus, the third-best arrangement must either be the capital-constrained arrangement or the long duration arrangement.

3.2.2 The third best

With the above analysis, we now only have two third-best candidates: the capital-constrained arrangement and the long duration arrangement. As the GP gives LPs no rents in either case, the investment surplus from the capital-constrained arrangement and that from the long duration arrangement are given by the GP's respective payoffs, i.e., given by (10) and (12) respectively. Comparing these two expressions reveals their ranks in terms of efficiency. The following lemma presents the result.

LEMMA 2 *There exists a critical value $\pi_{CL}(p,\lambda)$ (defined in the Appendix) such that the capital-constrained arrangement is the third-best arrangement if $R/I \leq \pi_{CL}(p,\lambda)$ and the long duration arrangement is the third-best arrangement if $R/I > \pi_{CL}(p,\lambda)$.*

Lemma 2 shows that, everything else being equal, the higher (lower) the projects' payoff potential, the more likely that the long duration arrangement has higher (lower) efficiency compared to the capital-constrained arrangement. This is intuitive. Compared to the capital-constrained arrangement, the long duration arrangement has both the advantage in terms of eliminating underinvestment and the disadvantage in terms of exacerbating overinvestment. Everything else being equal, the higher the projects' payoff potential, the larger the efficiency loss caused by underinvestment while the smaller the efficiency loss caused by overinvestment. Both effects tend to make the long duration arrangement more efficient than the capital-constrained arrangement.

As the third-best financing arrangement, no matter whether it is the long duration or the capital-constrained arrangement, offers LPs no rents, all the other less efficient financing arrangements will not be chosen by the GP in equilibrium. Thus, to determine the equilibrium financing arrangement, we only have to compare the GP's payoff from the second-best arrangement with his payoff from the third-best arrangement. We present the result in the next part of this section.

3.3 The equilibrium financing arrangement

The second-best arrangement is more efficient than the third-best but may offer LPs rents, while the third-best arrangement is rent-saving but less efficient than the second-best. In essence, the GP chooses between the second-best and the third-best arrangement by trading off between saving agency costs and saving rents. By Proposition 1, the second-best arrangement offers LPs rents only if projects' payoff potential is higher than some critical value, which is thus the necessary condition for the third-best arrangement to be adopted in equilibrium. The following theorem presents the main result of this paper. It shows that under certain parametric assumptions on p and λ, the capital-constrained arrangement is adopted in equilibrium if projects have mildly high payoff potential while the long duration arrangement is used in equilibrium if projects have sufficiently high payoff potential.

THEOREM 1 *Suppose* $2 < R/I < 1/p$. *The equilibrium financing arrangement has the following features:*

(i) Fixing p *and* λ, *as* $R/I \to 1/p$, *the GP either chooses the second-best arrangement or the long duration arrangement;*

(ii) Fixing p *and* λ, *if the GP chooses the second-best arrangement as* $R/I \to 1/p$, *then he chooses the second-best arrangement for all* $\pi_0(p, \lambda) \leq R/I < 1/p$, *where* $\pi_0(p, \lambda)$ *is defined by (22) in the Appendix;*

(iii) Fixing p *and* λ, *if the GP chooses the long duration arrangement as* $R/I \to 1/p$, *then there must either (a) exist one critical value* $\pi_{SL}(p, \lambda)$ *(defined in the Appendix) such that the GP prefers the second-best arrangement when* $\pi_0(p, \lambda) \leq R/I \leq \pi_{SL}(p, \lambda)$ *and prefers the long duration arrangement when* $\pi_{SL}(p, \lambda) < R/I < 1/p$ *or (b) exist two critical points* $\pi_{SC}(p, \lambda)$ *and* $\pi_{CL}(p, \lambda)$ *(defined in the Appendix), where* $\pi_{SC}(p, \lambda) < \pi_{CL}(p, \lambda)$, *such that the GP prefers the second-best arrangement when* $\pi_0(p, \lambda) \leq R/I \leq \pi_{SC}(p, \lambda)$, *prefers the capital-constrained arrangement when* $\pi_{SC}(p, \lambda) < R/I \leq \pi_{CL}(p, \lambda)$, *and prefers the long duration arrangement when* $\pi_{CL}(p, \lambda) < R/I < 1/p$.

Theorem 1 implies that the size and investment duration of a VC fund depend on project characteristics. Everything else being equal, the lower the projects' payoff potential, the more likely that the VC fund imposes investment duration restrictions. Moreover, everything else being equal, the GP is more likely to establish a larger (smaller) VC fund if the projects' payoff potential is either high or low (is moderate).

4 Extensions and Robustness

4.1 Investment focus and correlated project quality

In our model, we assumed independence of project quality. In practice, the GP's candidate projects are likely to be exposed to some common shocks, such as industry shocks. It is thus reasonable to think that the GP specializing in one industry faces more correlated projects compared to the GP with a lower investment focus. To model project quality correlation, we keep each project's unconditional quality the same as before, i.e., each project's unconditional probability of being good is still λ, and we allow each project's quality conditioned on the other project's quality to be different from its unconditional quality. Particularly, we denote by τ a project's probability of being good conditioned on the other project being good, where $\lambda \leq \tau \leq 1$. Notably, if we denote by τ' a project's probability of being good conditioned on the other project being bad, then the law of total probability implies the following:

$$\tau\lambda + \tau'(1-\lambda) = \lambda, \tag{15}$$

from which we obtain

$$\tau' = \left(\frac{1-\tau}{1-\lambda}\right)\lambda. \tag{16}$$

As $\lambda \leq \tau \leq 1$, by (16), $0 \leq \tau' \leq \lambda$. It is easy to see that project quality correlation increases as τ increases. When τ equals λ, τ' also equals λ and the two projects have independent quality. When τ equals 1, τ' equals 0 and the two projects have perfectly correlated quality. The next proposition shows that an increase in project quality correlation makes it less likely for the GP to use the capital-constrained arrangement.

Proposition 5 *Fixing p, λ, and R/I, if the capital constrained arrangement is not used when $\tau = \tau_0$, neither would it be used when $\tau > \tau_0$; if the capital constrained arrangement is used when $\tau = \tau_0$, there must exit $\bar{\tau}$, where $\tau_0 < \bar{\tau} < 1$, such that the capital constrained arrangement is never used for $\tau \geq \bar{\tau}$.*

The intuition of Proposition 5 is as follows. Compared to the long duration arrangement, the capital-constrained arrangement mitigates overinvestment in state BB while causes underinvestment in state GG. Because an increase in project quality correlation increases the probability of state GG and of state BB occurring by the same amount and also because, when $R > 2I$, the inefficiency from missing a good project is larger than the inefficiency from investing in a bad project, an increase in project quality correlation reduces the efficiency associated with the capital-constrained arrangement relative to that associated with the long duration arrangement. As the long duration arrangement offers LPs no rents, an increase in project quality correlation must tend to make the long duration arrangement more favorable to the GP compared to the capital-constrained arrangement. Compared to the second-best arrangement, the capital-constrained arrangement is always less efficient, since it causes underinvestment in state GG. As an increase in project quality correlation increases the probability of state GG occurring, it further reduces the efficiency associated with the capital-constrained arrangement relative to that associated with the second-best arrangement. Although the second-best arrangement may offer LPs rents, it turns out that an increase in project quality correlation never increases LPs' rents under the second-best arrangement by more than it increases the efficiency loss, under the capital-constrained arrangement, from missing one good project in state GG. Therefore, an increase in project quality correlation also tends to make the second-best arrangement more favorable to the GP compared to the capital-constrained arrangement.

Note that, it is ambiguous whether an increase in project quality correlation tends to make the second-best arrangement more favorable to the GP compared to the long duration arrangement. Although an increase in project quality correlation increases the efficiency loss associated with the long duration arrangement relative to that associated with the second-best arrangement, it can sometimes increase LPs' rents under the second-best arrangement by more than it increases the inefficiency of the long duration arrangement relative to the second-best, making the second-best arrangement less favorable to the GP compared to the long duration arrangement. We provide an example of this situation in the Appendix.

Proposition 5 implies that the GP with a higher investment focus tends to have a larger VC fund.

4.2 LPs' bargaining power

In our model, we assumed that the capital market is perfectly competitive, so LPs are willing to finance the GP if they can break even. Now we relax this assumption by offering LPs some bargaining power. We model the bargaining process as a two-period model of alternating offers with a risk of breakdown.[13] We assume that, at $t = 0$, to raise funds, the GP makes an offer to LPs. If LPs accept the offer, the game goes on to $t = 1$. However if LPs reject the offer, with probability $1 - \delta$, where $0 \leq \delta \leq 1$, negotiation breaks down and every player receives zero payoff, while with probability δ, negotiation goes on to $t = 0.5$, at which time LPs are allowed to make a counter-offer. If the GP accepts LPs' counter-offer, the game goes on to $t = 1$ and if otherwise, the game ends with every player receiving zero payoff.

Given the above simple structure of bargaining, it is easy to work out both parties' strategies. Suppose the second-best surplus is nonnegative so that financing is feasible. In that case, if LPs reject the GP's offer and are able to make a counter-offer, the offer that maximizes LPs' expected payoff would be a short duration arrangement with a contract that pledges (almost) all the marginal cash flows to LPs, since such an offer not only implements the second best but also gives all the second-best surplus to LPs. As LPs can only make a counter-offer with probability δ, their expected payoff by rejecting the GP's offer equals δ times the second-best surplus. Thus, the GP, when making his offer at $t = 0$, has to give LPs the amount of rents equal to δ times the second-best surplus in order to have LPs accept the offer. As regardless of what financing arrangement the GP adopts with his offer, the GP has to give LPs the same amount of rents, the disadvantage of the second-best arrangement in terms of rent-offering becomes less of a problem while its advantage in terms of efficiency-enhancing is unaffected by the imposed minimum rents the GP has to offer. Thus, increasing LPs' bargaining power tends to induce the GP to use the second-best arrangement. We present this result formally in the next proposition.

PROPOSITION 6 *Fixing p, λ, and R/I, if the second-best arrangement is used when $\delta = \delta_0$, it must also be used when $\delta > \delta_0$; if the second-best arrangement is not used when $\delta = \delta_0$, there must exist $\bar{\delta}$, where $\delta_0 < \bar{\delta} < 1$, such that the second-best arrangement is used for all $\delta \geq \bar{\delta}$.*

Proposition 6 implies that an increase in LPs' bargaining power increases the efficiency of VC investments.

[13]This simple formulation of bargaining is adopted by other papers, e.g., by Noe and Wang (2004). For a general discussion of the model of alternating offers with a risk of breakdown, see Osborne and Rubinstein (1990, Chapter 4.2).

4.3 $R/I \leq 2$

In our model, we assumed that $R/I > 2$. This assumption is reasonable since venture capital projects usually have high payoff potential. We also need this assumption to give the GP, in state BB under the long duration arrangement, the incentive to invest in the other bad project if he foresees the failure of his earlier investment. However, for the sake of completeness, here we investigate the case where $R/I \leq 2$. Although it turns out that this change in assumption would not overturn the qualitative nature of our previous results, the analysis here has its own merit: We show that, when $R/I \leq 2$, in state BB under the long duration arrangement, the GP has an incentive to invest in the other bad project if he foresees the *success* of his earlier investment.

The second best can be implemented by the second-best arrangement discussed before, that is, a short duration arrangement with the GP security satisfying condition (IC$_{BB}$). When $R/I \leq 2$, if the GP raises $2I$, by conditions (M) and (NP), the GP security must satisfy $s_{GP}(R) \leq s_{GP}(2I) = 0$. Thus, condition (IC$_{BB}$) becomes

$$s_{GP}(R + I) \geq p s_{GP}(2R). \tag{17}$$

The next proposition shows that, if $p \leq 1/2$, conditioned on that financing is feasible, the GP always uses the second-best arrangement.

PROPOSITION 7 *Suppose $R/I \leq 2 < 1/p$. There is a critical value $\pi_0(p, \lambda)$ (defined by* (22) *in the Appendix) such that*

(i) for projects with lower payoff potential (that is, $R/I < \pi_0(p, \lambda)$), financing is infeasible;

(ii) for projects with higher payoff potential (that is, $R/I \geq \pi_0(p, \lambda)$), the GP finances both projects by using the second-best arrangement with the optimal GP security s_{GP} (which is not always unique) given by

$$s_{GP}(x) = \begin{cases} 0 & \text{if } x \leq 2I \\ \alpha_S(x - 2I) & \text{if } x > 2I, \end{cases} \tag{18}$$

where $0 \leq \alpha_S \leq 1$ and α_S is determined by making the investors' break-even condition (5) *bind. By using this security under the second-best arrangement, the GP gives LPs no rents and the GP's expected payoff equals the second-best surplus given by* (5).

When $R/I \leq 2$ and $p \leq 1/2$, the GP incurs no rent costs by using the second-best arrangement and as the second-best arrangement is more efficient than alternative arrangements, it will always be preferred by the GP. However, the next proposition shows that when $p > 1/2$, implying $R/I < 2$, under certain parametric conditions, the GP uses the long duration arrangement if projects' payoff potential is higher.

PROPOSITION 8 *Suppose $R/I < 1/p < 2$. The equilibrium financing arrangement has the following features:*

(i) If

$$(1-\lambda)^2 p^2 - 2\lambda^2 p + \lambda^2 \geq 0, \tag{19}$$

the GP always chooses the second-best arrangement for all $\pi_0(p,\lambda) \leq R/I < 1/p$, where $\pi_0(p,\lambda)$ is defined by (22) in the Appendix;

(ii) If condition (19) is violated, there exists a critical value $\pi'_{SL}(p,\lambda)$ (defined in the Appendix) such that the GP chooses the second-best arrangement when $\pi_0(p,\lambda) \leq R/I \leq \pi'_{SL}(p,\lambda)$ and chooses the long duration arrangement when $\pi'_{SL}(p,\lambda) < R/I < 1/p$.

Note that both Proposition 7 and Proposition 8 satisfy the general features of the equilibrium financing arrangement presented by Theorem 1. Interestingly, when $R/I \leq 2$, if the GP wants to save rents by using arrangements other than the second-best, he would only use the long duration arrangement, and it is never optimal for the GP to have a smaller fund, i.e., using the capital-constrained arrangement. Note that this result only requires $R/I \leq 2$; it does not require any parametric assumptions imposed on p or λ. Similar to what we found before, the GP never uses the stand-alone arrangement or the non-second-best short duration arrangement. This is again because the long duration arrangement is rent-saving and more efficient than both the stand-alone and the non-second-best short duration arrangement. Interestingly and maybe surprisingly, the reason for the long duration arrangement to be more efficient than the stand-alone and the non-second-best short duration arrangement becomes different from what we found before. Under the long duration arrangement, in state BB, the GP, in earlier sections with $R/I > 2$, abandons the second bad project if his earlier investment *succeeds*, but now, with $R/I \leq 2$, he abandons the second bad project if his earlier investment *fails*, since now he cannot create an overall profit for the fund even if the second bad project succeeds.

5 Empirical Implications

As in our model, the short duration arrangement is distinguished from the long duration arrangement by the use of investment duration restrictions, before we present empirical implications of our model, it is necessary for us to discuss how to restrict VC investment duration in practice. Although the U.S. VC funds uniformly have a lifespan of 10-13 years, as discussed in the introduction, it does not mean that they offer the GP the same time period for the investment stage. In contrast, VC limited partnerships vary largely in their contractual terms and provisions, which results in different levels of investment duration restrictions. We think of a VC limited partnership as restricting investment duration if its agreement owns at least one of the following two features:

(a) the agreement bans investing into new companies after a certain period from the start of the partnership;

(b) the agreement includes both the "call-to-invest" provision and the "use-it-or-lose-it" provision.

In practice, (a) is the explicit way of restricting investment duration of a VC fund while (b) is an implicit way which needs more explanation. With a "call-to-invest" provision, the GP does not obtain the entire LPs' committed capital right after the establishment of the partnership, but has to make capital calls whenever he embarks on a project and the GP can only call as much capital as he can promptly invest. If the VC fund imposes this provision in our model, the GP can only call for I at $t = 1$ if he only invests in one project at $t = 1$ and he has to call for the other I at $t = 2$ if he wants to invest in the other project at $t = 2$. But in this case, as LPs anticipate that the later project has bad quality, they would not offer the GP the other I. Although in practice, to prevent LPs from defaulting on contributing capital, the limited partnership usually imposes default penalties on defaulters,[14] the refusal to contribute capital would not be considered as LP defaults if the agreement has a "use-it-or-lose-it" provision, which provides that LPs are not required to make capital contributions after a certain date (see Litvak 2004), e.g., after the start of $t = 2$ in our model. Thus, by including both the "call-to-invest" and the "use-it-or-lose-it" provision, the partnership

[14] Typically, a defaulter loses a portion of its interest in a VC fund (see Litvak 2004).

implicitly prevents the GP from making new investments at $t = 2$ and investment duration is thus shortened.

Our model generates a list of interesting and testable predictions.

(i) *Projects undertaken earlier by a VC fund are more profitable compared to those undertaken later by the same fund.* In our model, when the GP has long investment duration, he has no incentive to postpone investing in good projects, while for bad projects, he has an incentive to delay (part of) his investment decisions until he obtains new information regarding his earlier investments. This coincides with the empirical finding of Dass, Hsu, Nanda, and Wang (2012) that a VC fund's percentage of successful exists, through either IPO or merger and acquisitions, is much lower in later investments than in earlier investments.

(ii) *The larger the VC fund, the more likely that the partnership agreement restricts the fund's investment duration.* In our analysis, there always exists a positive probability that the GP's projects are all negative NPV gambles. A larger size of the VC fund offers the GP more chances to gamble with the money. Because of joint liability between the gambles, the information on the success/failure of part of his gambles is beneficial for the GP to decide whether to gamble more/less with the rest of the money. Thus, the GP has an incentive to play strategic sequential timing of investments, which may lead to multiple gambles played by the GP. To mitigate this problem, the limited partnership agreement may restrict the investment duration either through an explicit prohibition on investing into new projects after a certain date or through the "call-to-invest" and the "use-it-or-lose-it" provision. As pointed out by Kandel, Leshchinskii, and Yuklea (2011), "recently, many funds have begun to contractually restrict the search stage to 5-6 years, after which the GP can no longer make new investments." It would be interesting to examine empirically in the future whether such restrictions are positively correlated with the fund size.

(iii) *LPs earn more on average if investment duration restrictions are used.* Our analysis shows that restricting investment duration is only necessary but not sufficient to minimize agency conflicts; GP compensation must also provide the GP with sufficient incentives to abandon as many bad projects as possible. In this sense, partnership provisions that restrict investment duration and proper GP compensation are complements rather than substitutes. As such GP compensation may offer LPs rents, *ex ante* the GP trades off between saving agency costs and saving rents by choosing between the more efficient but possibly rent-offering financing arrangement and the less efficient but rent-saving arrangement. As in equilibrium, depending on parametric assumptions, either arrangement could be adopted, our model may help explain why investment duration restrictions are used by some VC funds but not by all. In addition, our model shows that LPs may earn rents when investment duration restrictions are imposed. This result may shed some light on the puzzling finding in Kaplan and Schoar (2005) that successful GPs do not seem to increase their fees in follow-up VC funds enough to force LPs down to a competitive rent. Whether LPs' rents are correlated with investment duration restrictions would be an interesting empirical question.

(iv) *VC funds specializing in traditional industries are more likely to impose investment duration restrictions than VC funds specializing in high-tech industries.* Our model establishes that, all else being equal, the lower the projects' payoff potential, the more likely the VC fund restricts the GP's investment duration. The prediction thus follows, as projects in traditional industries usually have a lower payoff potential compared to projects in high-tech industries.

(v) *VC funds are larger if projects' payoff potential is either high or low but small if it is moderate.* Our model shows that, on the one hand, if projects' payoff potential is low, a larger fund with investment duration restrictions would not only minimize agency costs but also give no or few rents to LPs, in which case reducing the VC fund size would be suboptimal for the GP. On the other hand, if projects' payoff potential is high, a smaller size of the VC fund would lead to large efficiency loss due to underinvestment, in which case it is better for the GP to establish a larger VC fund with no investment duration restrictions. Limiting the VC fund size is beneficial to the GP only if projects' payoff potential is moderate.

(vi) *The higher the investment focus of the GP, the larger the VC fund.* This prediction is implied by Proposition 5. This is, to our knowledge, a new and testable implication. To test this implication, it is important to distinguish between the investment focus of a GP and the investment focus of a VC fund. The investment focus of a GP refers to the GP's *ex ante* attribute, i.e., whether the GP specializes in one industry or several industries. In contrast, the investment focus of a VC fund is the fund's *ex post* attribute, i.e., whether the VC fund invests in one industry or several industries. It is noticeable that a smaller fund can only have a smaller portfolio, in which case it is plausible that a smaller fund has a higher *ex post* investment focus. But this does not mean that the *ex ante* investment focus of the GP who manages this smaller fund is also high. Surprisingly, our result implies the opposite: it is more likely for a GP with a lower investment focus to manage a smaller fund.

(vii) *The less competitive the capital market, the larger the VC fund and the more restrictive the investment timing.* This prediction is implied by Proposition 6. Recall that agency costs are minimized if the VC fund is large and if investment duration restrictions are imposed. Thus, this prediction suggests that making the capital market less competitive may benefit efficiency.

6 Conclusion

This paper studies the size and investment duration of a VC fund based on simple agency conflicts between the GP and LPs. We have identified four conditions to minimize agency conflicts: (i) larger fund size, (ii) cross-pledging, (iii) investment duration restrictions, and (iv) the GP's profit sharing limit. The first condition is necessary to contract off underinvestment while all the other conditions are useful to minimize overinvestment. However, to satisfy the fourth condition, the contract may offer LPs positive rents even if the capital market is perfectly competitive. If the rents offered to LPs are too large, which occurs when projects have higher payoff potential, the GP trades off investment efficiency for reduced rents by using less efficient but rent-saving financing arrangements. One such an alternative financing arrangement is to limit the size of the fund, which causes underinvestment. A smaller fund is chosen only if projects have moderate payoff potential. The other alternative financing arrangement is to keep the fund size large but impose no investment duration restrictions. Under this arrangement, the GP has an incentive to play strategic sequential timing of investments that exacerbates overinvestment. A larger fund with no investment duration restrictions is chosen only if projects have sufficiently high payoff potential.

As the GP usually invests in startups that have high payoff potential, according to our theory, it is not surprising that many VC funds have large fund size that enables the GP to invest in many projects and do not restrict investment duration heavily. Such an arrangement induces the GP to play strategic sequential timing of investments, which gives the GP the chance of collecting *ex post* information on the progress of earlier investments, and such information is used by the GP solely for the purpose of "gaming" the contract at the expense of efficiency. Contracting off strategic sequential timing of investments requires the limited partnership to contractually restrict the GP's investment duration. One important message of the paper is that restricting investment duration *per se* cannot guarantee an improvement in efficiency; if GP compensation is not properly designed, the overinvestment problem could even be *exacerbated* by restricting investment duration. This may serve as a caveat to VC funds that intend to restrict investment duration. As a proper security design may offer LPs positive rents that increases the GP's financing costs, the GP may choose less efficient financing arrangements although he bears the efficiency loss *ex ante*. One useful implication from our model is that, to induce the GP to choose the more efficient financing arrangement, it would be helpful to increase LPs' bargaining power, since in that case the GP has to offer LPs positive rents anyway and thus the disadvantage of the more efficient financing arrangement, in terms of rent-offering, becomes less of a problem.

Appendix A: Proofs

Outline of proof of Proposition 1: We restate the GP's maximization problem in Section 3.1.1 as follows:

$$\max_{s_{GP}(x)} E[s_{GP}(x)] = \lambda^2 s_{GP}(2R) + \left[2\lambda(1-\lambda) + (1-\lambda)^2 p\right] s_{GP}(R+I), \tag{20}$$

such that

$$\begin{aligned} E[x - s_{GP}(x)] &\geq 2I && \text{(BE)} \\ p s_{GP}(R+I) &\geq p^2 s_{GP}(2R) + 2p(1-p) s_{GP}(R) && \text{(IC}_{BB}\text{)} \\ s_{GP}(x) &= 0 \qquad \forall x \quad s.t.\ x \leq 2I && \text{(NP)} \\ x - x' &\geq s_{GP}(x) - s_{GP}(x') \qquad \forall x, x' \quad s.t.\ x > x' && \text{(M)} \end{aligned}$$

By using the second-best arrangement, the GP invests only in good project(s) in states GG and GB and invests in one bad project in state BB. Such investment behavior leads to the second-best surplus equal to

$$\lambda^2(2R - 2I) + 2\lambda(1-\lambda)(R - I) + (1-\lambda)^2(pR - I). \tag{21}$$

Rearranging (21) gives (5). Since investors must at least break even, (21) has to be nonnegative for the GP to raise funds. Thus, the second-best arrangement is feasible if and only if $R/I \geq \pi_0(p, \lambda)$, where $\pi_0(p, \lambda)$ is defined as

$$\pi_0(p, \lambda) \equiv \frac{1 + \lambda^2}{2\lambda + (1-\lambda)^2 p}. \tag{22}$$

As the GP security has to satisfy (IC_{BB}), it may offer LPs rents and thus the GP may not earn the second-best surplus. Below we solve the program by first solving the program with the investors' break-even condition (BE) omitted. Lemmas 3 and 4 offer some characterizations of the GP's optimal security that are useful for solving the program. Based on these two lemmas, Lemmas 5, 6, 7, 8, 9 characterize the GP's maximum expected payoff without (BE). Finally, we take into account (BE): the GP's actual expected payoff is the minimum between the GP's maximum expected payoff without (BE) and the second-best surplus. The expression for $\pi_{rent}(p, \lambda)$ and the GP's expected payoff when R/I exceeds $\pi_{rent}(p, \lambda)$ are provided by the proof.

LEMMA 3 *Holding the expected payment to investors fixed, it is without loss of generality to set $s_{GP}(R + I) - s_{GP}(R)$ as high as possible: either $s_{GP}(R + I) - s_{GP}(R) = I$, $s_{GP}(2R) \geq s_{GP}(R + I)$, and $s_{GP}(R) \geq 0$, or $s_{GP}(R + I) - s_{GP}(R) < I$, $s_{GP}(2R) = s_{GP}(R + I)$, and $s_{GP}(R) = 0$.*

Proof: We first show that given $s_{GP}(R)$, increasing $s_{GP}(R + I)$, which is equivalent to increasing $s_{GP}(R + I) - s_{GP}(R)$, while decreasing $s_{GP}(2R)$ such that the maximand and the expected payment to investors are held constant will relax (IC_{BB}). Therefore, as long as conditions (LL), (M), and (NP) are not violated, this will relax the program. Note that when $s_{GP}(R + I) = s_{GP}(R) + I$ (the maximum payment given $s_{GP}(R)$) or when $s_{GP}(2R) = s_{GP}(R + I)$, we cannot do this perturbation without violating (M).

Given $s_{GP}(R)$, increasing $s_{GP}(R + I)$ while decreasing $s_{GP}(2R)$ such that the maximand

and the expected payment to investors are held constant requires

$$-ds_{GP}(2R) = \frac{2\lambda(1-\lambda)+(1-\lambda)^2 p}{\lambda^2} ds_{GP}(R+I). \tag{23}$$

Obviously, this perturbation relaxes (IC$_{BB}$).

Next it is clear that decreasing $s_{GP}(R)$ while increasing $s_{GP}(R+I) - s_{GP}(R)$ by the same amount keeps $s_{GP}(R+I)$ and $s_{GP}(2R)$ constant. Hence this perturbation keeps the maximand and the expected payment to investors constant. Note that when $s_{GP}(R) = 0$ or when $s_{GP}(R+I) - s_{GP}(R) = I$, we cannot do this perturbation without violating (M). As this perturbation decreases $s_{GP}(R)$ while keeps $s_{GP}(R+I)$ and $s_{GP}(2R)$ fixed, this perturbation relaxes (IC$_{BB}$). $\square$

LEMMA 4 *Given $s_{GP}(R+I) - s_{GP}(R) = I$ and fixing the expected payment to investors, when*

$$p \geq \frac{\lambda}{\sqrt{1+(1-\lambda)^2}+1}, \tag{24}$$

it is without loss of generality to set $s_{GP}(2R) - s_{GP}(R+I)$ as low as possible: either $s_{GP}(2R) - s_{GP}(R+I) = 0$ and $s_{GP}(R+I) < R-I$, or $s_{GP}(2R) - s_{GP}(R+I) \geq 0$ and $s_{GP}(R+I) = R-I$.

If (24) does not hold, it is without loss of generality to set $s_{GP}(2R) - s_{GP}(R+I)$ as high as possible: either $s_{GP}(2R) - s_{GP}(R+I) = R-I$ and $s_{GP}(R+I) \geq I$, or $s_{GP}(2R) - s_{GP}(R+I) < R-I$ and $s_{GP}(R+I) = I$.

Proof: We show that given $s_{GP}(R+I) - s_{GP}(R)$, when (24) holds, increasing $s_{GP}(R)$ while decreasing $s_{GP}(2R)$ such that the maximand and the expected payment to investors are held constant will relax (IC$_{BB}$). Therefore, as long as (LL), (M), and (NP) are not violated, this perturbation will relax the program. Note that when $s_{GP}(R) = R - 2I$ or when $s_{GP}(2R) = s_{GP}(R+I)$, we cannot do this perturbation without violating (M).

Given $s_{GP}(R+I) - s_{GP}(R)$, increasing $s_{GP}(R)$ and decreasing $s_{GP}(2R)$ such that the payment to investors and the maximand are held constant will lead to (23) and $ds_{GP}(R) = ds_{GP}(R+I)$. Moving all terms in (IC$_{BB}$) to its left-hand side, the change in the left-hand side as we fix $s_{GP}(R+I) - s_{GP}(R)$, increase $s_{GP}(R)$, and decrease $s_{GP}(2R)$ is equal to

$$\begin{aligned} & p\,ds_{GP}(R+I) - p^2 ds_{GP}(2R) - 2p(1-p)ds_{GP}(R) \\ = \; & p\left[1 + p\frac{2\lambda(1-\lambda)+(1-\lambda)^2 p}{\lambda^2} - 2(1-p)\right] ds_{GP}(R) \\ = \; & \frac{p}{\lambda^2}\left[-\lambda^2 + 2\lambda p + (1-\lambda)^2 p^2\right] ds_{GP}(R). \end{aligned} \tag{25}$$

When (24) holds, (25) is greater than zero if $ds_{GP}(R) > 0$. In other words, when (24) holds, the above perturbation relaxes (IC_{BB}).

When (24) does not hold, (25) is greater than zero if $ds_{GP}(R) < 0$. In other words, when (24) does not hold, given $s_{GP}(R+I) - s_{GP}(R)$, decreasing $s_{GP}(R)$ while increasing $s_{GP}(2R)$ such that the payment to investors and the maximand are held constant will relax (IC_{BB}). Note that when $s_{GP}(R) = 0$ or when $s_{GP}(2R) = s_{GP}(R+I) + R - I$, we cannot do this perturbation without violating (M). □

LEMMA 5 *If* (24) *holds and* $R/I \leq 3 - 2p$, *with the investors' break-even condition omitted, the GP security which offers the GP the maximum expected payoff without violating* (IC_{BB}) *has* $s_{GP}(R) = R - 2I$, $s_{GP}(R+I) = R - I$, *and* $s_{GP}(2R) = 2R - 2I$. *This security offers the GP the expected payoff equal to*

$$\Delta_{GP} = \left[2\lambda + (1-\lambda)^2 p\right](R - I). \tag{26}$$

Proof: By Lemmas 3 and 4, when (24) holds, fixing the expected payment to investors, it is without loss of generality to set $s_{GP}(R+I) - s_{GP}(R)$ as high as possible, and given $s_{GP}(R+I) - s_{GP}(R) = I$, to set $s_{GP}(2R) - s_{GP}(R+I)$ as low as possible. In other words, it is without loss of generality to pledge to the GP the marginal cash flows within the region $[R, R+I]$ first, then the marginal cash flows within the region $[2I, R]$, and finally the marginal cash flows within the region $[R+I, 2R]$. When $R/I \leq 3 - 2p$, it is easy to check that the GP security stated in Lemma 5 satisfies (IC_{BB}). Since this security pledges all the marginal cash flows within the region $[2I, 2R]$ to the GP, it is impossible to further increase the GP's payoff without violating (LL), (M), or (NP). By using the security stated in Lemma 5, the GP's expected payoff, expressed by (20), equals (26). □

LEMMA 6 *If* (24) *holds and* $3 - 2p < R/I \leq 3$, *with the investors' break-even condition omitted, the GP security which offers the GP the maximum expected payoff without violating* (IC_{BB}) *has* $s_{GP}(R) = R - 2I$, $s_{GP}(R+I) = R - I$, *and* $s_{GP}(2R) = R - I + (3I - R)(1-p)/p$. *This security offers the GP the expected payoff equal to*

$$\Delta_{GP} = \left[2\lambda + (1-\lambda)^2 p - \frac{\lambda^2}{p}\right] R - \left[2\lambda + 2\lambda^2 + (1-\lambda)^2 p - \frac{3\lambda^2}{p}\right] I. \tag{27}$$

Proof: When (24) holds, the optimal way of pledging marginal cash flows to the GP without violating (IC_{BB}) is stated in the proof of Lemma 5. It is clear that the GP security presented in Lemma 6 makes (IC_{BB}) bind. Since this security pledges all the marginal cash flows within the region $[2I, R+I]$ to the GP, the only way to further increase the GP's maximum expected payoff is by increasing the GP's share of the marginal cash flows within the region $[R+I, 2R]$, but this will violate (IC_{BB}). By using the security stated in Lemma 6, the GP's expected payoff, expressed by (20), equals (27). □

LEMMA 7 *If* (24) *holds and* $R/I > 3$*, with the investors' break-even condition omitted, the GP security which offers the GP the maximum expected payoff without violating* (IC_{BB}) *has* $s_{GP}(R) = I$ *and* $s_{GP}(R+I) = s_{GP}(2R) = 2I$*. This security offers the GP the expected payoff equal to*

$$\Delta_{GP} = \left[4\lambda - 2\lambda^2 + 2(1-\lambda)^2 p\right] I. \tag{28}$$

Proof: When (24) holds, the optimal way of pledging marginal cash flows to the GP without violating (IC_{BB}) is stated in the proof of Lemma 5. When $R/I > 3$, it is easy to check that the GP security stated in Lemma 7 makes (IC_{BB}) bind. Since this security pledges all the marginal cash flows within the region $[R, R+I]$ to the GP and part of the marginal cash flows within the region $[2I, R]$ to the GP, further increasing the GP's payoff requires an increase in the GP's share of the marginal cash flows within the region $[2I, R]$, but this will violate (IC_{BB}). By using the security stated in Lemma 7, the GP's expected payoff, expressed by (20), equals (28). □

LEMMA 8 *If* (24) *does not hold and if* $R/I \leq 3 - 2p$*, with the investors' break-even condition omitted, the GP security which offers the GP the maximum expected payoff without violating* (IC_{BB}) *is the same as the one presented in Lemma 5. This security gives the GP the expected payoff equal to* (26)*.*

Proof: If (24) does not hold, by Lemmas 3 and 4, it is without loss of generality to pledge to the GP the marginal cash flows within the region $[R, R+I]$ first, then the marginal cash flows within the region $[R+I, 2R]$, and finally the marginal cash flows within the region $[2I, R]$. When $R/I \leq 3 - 2p$, it is easy to check that the GP security stated in Lemma 5 satisfies (IC_{BB}). Since this security pledges all the marginal cash flows within the region $[2I, 2R]$ to the GP, it is impossible to further increase the GP's payoff without violating (LL), (M), or (NP). By using the security stated in Lemma 8, the GP's expected payoff, expressed by (20), equals (26). □

LEMMA 9 *If (24) does not hold and if $R/I > 3-2p$, with the investors' break-even condition omitted, the GP security which offers the GP the maximum expected payoff without violating* (IC_{BB}) *has* $s_{GP}(R) = (I-pR)/(1-p)$, $s_{GP}(R+I) = (I-pR)/(1-p)+I$, *and* $s_{GP}(2R) = (I-pR)/(1-p)+R$. *This security offers the GP the expected payoff equal to*

$$\Delta_{GP} = \left[1+\lambda^2+(1-\lambda)^2p-\frac{1}{1-p}\right]R+\left[(1-\lambda)(3\lambda-1)+(1-\lambda)^2p+\frac{1}{1-p}\right]I. \quad (29)$$

Proof: If (24) does not hold, the optimal way of pledging marginal cash flows to the GP is stated in the proof of Lemma 8. It is clear that the security stated in Lemma 9 makes (IC_{BB}) bind. As this security pledges all the marginal cash flows in the region $[R+I, 2R]$ to the GP and part of the marginal cash flows in the region $[2I, R]$ to the GP, further increasing the GP's payoff requires an increase in the GP's share of the marginal cash flows within the region $[2I, R]$, but this will violate (IC_{BB}). By using the security stated in Lemma 9, the GP's expected payoff, expressed by (20), equals (29). □

Proof of Proposition 1: Lemmas 5, 6, 7, 8, and 9 present the GP's maximum expected payoff, Δ_{GP}, in the program without (BE). If (BE) is further imposed, the GP's expected payoff cannot exceed the second-best surplus given by (5). Since the GP maximizes his expected payoff, in each of the situations presented in Lemmas 5, 6, 7, 8, and 9, the GP's expected payoff equals the minimum of Δ_{GP} stated in the respective lemma and the second-best surplus. The critical value π_{rent} can be derived by equalizing Δ_{GP} and the second-best surplus. For the sake of brevity, in what follows, we display the result without presenting the detailed calculation.

Note that in all the following cases, the GP obtains the second-best surplus expressed by (5) if and only if $\pi_0(p,\lambda) \leq R/I \leq \pi_{rent}(p,\lambda)$, where $\pi_0(p,\lambda)$ is defined by (22) while the expression for $\pi_{rent}(p,\lambda)$ varies case by case.

(i) when

$$p < \frac{\lambda}{1+\sqrt{(1-\lambda)^2+1}}, \quad (30)$$

$\pi_{rent}(p,\lambda)$ is given by

$$\pi_{rent}(p,\lambda) = 2-p+\frac{1-p}{2\lambda-\lambda^2+(1-\lambda)^2p}. \quad (31)$$

If R/I exceeds $\pi_{rent}(p,\lambda)$, the GP's payoff is given by (29).

(ii) when

$$p \geq \max\left\{\frac{\lambda}{1+\sqrt{(1-\lambda)^2+1}}, \frac{1-2\lambda-\lambda^2}{(1-\lambda)^2}\right\}, \tag{32}$$

$\pi_{rent}(p, \lambda)$ is given by

$$\pi_{rent}(p, \lambda) = \frac{3\lambda^2 + (1-2\lambda-\lambda^2)p - (1-\lambda)^2 p^2}{\lambda^2}. \tag{33}$$

If $\pi_{rent}(p, \lambda) \leq R/I \leq 3$, the GP's payoff is given by (27), while if $R/I > 3$, the GP's payoff is given by (28).

(iii) when

$$\frac{\lambda}{1+\sqrt{(1-\lambda)^2+1}} < p < \min\left\{\frac{1}{2}, \frac{1-2\lambda-\lambda^2}{(1-\lambda)^2}\right\}, \tag{34}$$

$\pi_{rent}(p, \lambda)$ is given by

$$\pi_{rent}(p, \lambda) = 2 + \frac{1-\lambda^2}{2\lambda + (1-\lambda)^2 p}. \tag{35}$$

If R/I exceeds $\pi_{rent}(p, \lambda)$, the GP's payoff is given by (28). □

Proof of Proposition 2: The investment surplus under the capital-constrained arrangement equals expression of (10). Obviously, the contract given by (9) satisfies (LL), (M), and (NP). LPs' expected payoff is strictly decreasing in α_C. When $\alpha_C = 0$, LPs' expected payoff equals the surplus (10). When $\alpha_C = 1$, LPs' expected payoff is negative. Thus, when investments under the capital-constrained arrangement are economically viable, i.e., when (10) is nonnegative, there always exists a unique α_C such that LPs just break even under the capital-constrained arrangement. Finally, (10) is nonnegative if and only if

$$\frac{R}{I} \geq \pi_C(p, \lambda), \tag{36}$$

where $\pi_C(p, \lambda)$ is defined as

$$\pi_C(p, \lambda) \equiv \frac{1}{2\lambda - \lambda^2 + (1-\lambda)^2 p}. \quad \square \tag{37}$$

Proof of Lemma 1: Suppose the GP uses a long duration arrangement with $s_{GP}(R) = 0$. We first prove that, in that case, if the second-best surplus is nonnegative and thereby financing is feasible, it would be optimal for the GP to implement the second best by using the following GP security:

$$s_{GP}(x) = \begin{cases} 0 & \text{if } x \leq R \\ \alpha(x - R) & \text{if } x > R, \end{cases} \tag{38}$$

where $0 \leq \alpha \leq 1$ and α equals 1 if the investors' break-even condition (5) holds with $\alpha = 1$ and is determined by making condition (5) bind if otherwise.

Note that the above security satisfies both (IC^{Δ}_{BB}) and (IC^{Δ}_{GB}), so by using the above security under the long duration arrangement, the GP creates the second-best surplus and his payoff equals the minimum of the second-best surplus and the expected payoff by using the security in the form of (38) with $\alpha = 1$. Note that given $s_{GP}(R) = 0$, the security in the form of (38) with $\alpha = 1$ pledges to the GP all the marginal cash flows that can possibly be pledged to him. Therefore, the GP cannot earn an expected payoff greater than that. In addition, the GP cannot earn an expected payoff greater than the second-best surplus. Hence, under the long duration arrangement with $s_{GP}(R) = 0$, the GP maximizes his payoff by using the GP security in the form of (38).

Next we show that the above arrangement gives the GP an expected payoff (weakly) less than that offered by the second-best (short duration) arrangement. To implement the second best under the short duration arrangement, the GP security has to satisfy conditions (IC_{GB}) and (IC_{BB}). Note that the above security (38) satisfies both (IC_{GB}) and (IC_{BB}), which implies that implementing the second best by using the above security under the short duration arrangement gives the GP the expected payoff equal to his maximum expected payoff under the long duration arrangement with $s_{GP}(R) = 0$. In addition, note that to implement the second best under the short duration arrangement, it is sufficient but not necessary to use the above security (38). This implies that the GP can possibly do better, in terms of rent-saving, by implementing the second best under the short duration arrangement rather than under the long duration arrangement. $\square$

Proof of Proposition 3: The investment surplus under the long duration arrangement with the GP security satisfying (IC_{GB}) equals the expression of (12). It is evident that the contract expressed by (11) satisfies (LL), (M), (NP), and (IC_{GB}). Note that LPs' expected payoff is strictly decreasing in α_L. When $\alpha_L = 0$, LPs' expected payoff equals the investment surplus under the long duration arrangement expressed as (12). When $\alpha_L = 1$, LPs' expected payoff is negative. Thus, when investments under the long duration arrangement are economically viable, i.e., when (12) is nonnegative, there always exists a unique α_L such that LPs just break even under the long duration arrangement. Finally, (12) is nonnegative if and only if

$$\frac{R}{I} \geq \pi_L(p, \lambda), \tag{39}$$

where $\pi_L(p, \lambda)$ is defined as

$$\pi_L(p, \lambda) \equiv \frac{2 - 2\lambda(1-\lambda) - (1-\lambda)^2 p}{2\lambda + (1-\lambda)^2(2-p)p}. \quad \square \tag{40}$$

Proof of Lemma 2: Comparing (10) and (12) shows that (10) is larger if and only if $R/I \leq \pi_{CL}(p, \lambda)$, where

$$\pi_{CL}(p, \lambda) = \frac{\lambda^2 + (1-\lambda)^2 - (1-\lambda)^2 p}{\lambda^2 + (1-\lambda)^2 p - (1-\lambda)^2 p^2}, \tag{41}$$

while (12) is larger if and only if $R/I \geq \pi_{CL}$. $\square$

Proof of Theorem 1: To find out the equilibrium financing arrangement, we simply need to compare between the GP's payoff from the second-best arrangement, enclosed in the proof of Proposition 1, his payoff from the capital-constrained arrangement, presented in Proposition 2, and his payoff from the long duration arrangement, presented in Proposition 3. The difficulty purely lies in the algebra as the GP's payoff from the second-best arrangement takes different functional forms under different parametric assumptions. For the sake of brevity, in what follows, we present the result without detailed calculation.

- Suppose (30) holds.
  - If
    $$\frac{1}{p} \leq 2 - p + \frac{1-p}{2\lambda - \lambda^2 + (1-\lambda)^2 p}, \tag{42}$$
    then the second-best arrangement will always be preferred by the GP.
  - If (42) does not hold,
    * suppose
      $$\pi_{CL}(p, \lambda) \leq \frac{2 + 2\lambda - \lambda^2 - (1 + 2\lambda - \lambda^2)p}{1 - (1-\lambda)^2(1-p)(1-p+p^2)}, \tag{43}$$
      where the critical value $\pi_{CL}(p, \lambda)$ is defined in (41), then there exists a critical value $\pi_{SL}(p, \lambda)$ expressed by the righthand side of (43) such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p, \lambda)$ while prefers the long duration arrangement if otherwise;
    * suppose (43) does not hold, then there exist critical values $\pi_{CL}(p, \lambda)$ and $\pi_{SC}(p, \lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SC}(p, \lambda)$, prefers the capital-constrained arrangement if $\pi_{SC}(p, \lambda) < R/I <$

$\pi_{CL}(p,\lambda)$, and prefers the long duration arrangement if $R/I \geq \pi_{CL}(p,\lambda)$, where the critical value $\pi_{CL}(p,\lambda)$ is defined in (41) while the critical value $\pi_{SC}(p,\lambda)$ is expressed by

$$\pi_{SC}(p,\lambda) = \frac{1+4\lambda-3\lambda^2+(1-6\lambda+4\lambda^2)p-(1-\lambda)^2p^2}{2\lambda-2\lambda^2+(1-2\lambda+2\lambda^2)p}. \tag{44}$$

- Suppose (32) holds.

  – If

  $$\frac{1}{p} \leq \frac{3\lambda^2+(1-2\lambda-\lambda^2)p-(1-\lambda)^2p^2}{\lambda^2}, \tag{45}$$

  then the second-best arrangement will always be preferred by the GP.

  – If

  $$\frac{3\lambda^2+(1-2\lambda-\lambda^2)p-(1-\lambda)^2p^2}{\lambda^2} < \frac{1}{p} \leq 3, \tag{46}$$

  then there exists a critical value $\pi_{SL}(p,\lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p,\lambda)$ while prefers the long duration arrangement if otherwise, where the critical value $\pi_{SL}(p,\lambda)$ is expressed by

  $$\pi_{SL}(p,\lambda) = \frac{2-4\lambda-2(1-\lambda)^2p+\frac{3\lambda^2}{p}}{(1-\lambda)^2(1-p)p+\frac{\lambda^2}{p}}. \tag{47}$$

  – If $1/p > 3$,

    * suppose

    $$\frac{2-4\lambda-2(1-\lambda)^2p+\frac{3\lambda^2}{p}}{(1-\lambda)^2(1-p)p+\frac{\lambda^2}{p}} \leq 3, \tag{48}$$

    then there exists a critical value $\pi_{SL}(p,\lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p,\lambda)$ while prefers the long duration arrangement if otherwise, where the critical value $\pi_{SL}(p,\lambda)$ is given by (47);

    * suppose (48) does not hold, then there exists a critical value $\pi_{SL}(p,\lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p,\lambda)$ while prefers the long duration arrangement if otherwise, where the critical value $\pi_{SL}(p,\lambda)$ is expressed by

    $$\pi_{SL}(p,\lambda) = \frac{2+2\lambda+(1-\lambda)^2p}{2\lambda+2(1-\lambda)^2p-(1-\lambda)^2p^2}. \tag{49}$$

- Suppose (34) holds.

– If

$$\frac{1}{p} \leq 2 + \frac{1-\lambda^2}{2\lambda + (1-\lambda)^2 p}, \tag{50}$$

then the second-best arrangement will always be preferred by the GP.

– If (50) does not hold,

  * suppose

$$\frac{2 + 2\lambda + (1-\lambda)^2 p}{2\lambda + 2(1-\lambda)^2 p - (1-\lambda)^2 p^2} \leq \frac{1 + 4\lambda - 2\lambda^2 + 2(1-\lambda)^2 p}{2\lambda - \lambda^2 + (1-\lambda)^2 p}, \tag{51}$$

  then there exists a critical value $\pi_{SL}(p, \lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p, \lambda)$ while prefers the long duration arrangement if otherwise, where $\pi_{SL}(p, \lambda)$ is expressed by (49);

  * suppose (51) does not hold, then there exist two critical values $\pi_{SC}(p, \lambda)$ and $\pi_{CL}(p, \lambda)$ such that the GP prefers the second-best arrangement if $R/I \leq \pi_{SC}(p, \lambda)$, prefers the capital-constrained arrangement if $\pi_{SC}(p, \lambda) < R/I < \pi_{CL}(p, \lambda)$, and prefers the long duration arrangement if $R/I \geq \pi_{CL}(p, \lambda)$, where $\pi_{SC}(p, \lambda)$ is given by the righthand side of (51) and $\pi_{CL}(p, \lambda)$ is given by (41).

The above results imply the features of the equilibrium financing arrangement presented by Theorem 1. □

Proof of Proposition 5: Let ρ_S, ρ_C, and ρ_L denote the surplus from the second-best, from the capital-constrained, and from the long duration arrangement, respectively. As the long duration arrangement and the capital-constrained arrangement both offer LPs zero rents, the GP's expected payoff by using the long duration arrangement and that by using the capital-constrained arrangement equal ρ_L and ρ_C, respectively. Note that the GP's payoff under the second-best arrangement is the minimum of ρ_S and his payoff by using the GP security that implements the second best with investors' break-even condition (BE) omitted and maximizes his payoff. Let Δ'_{GP} denote the latter. Thus, the GP's payoff by using the second best arrangement equals $\min\left[\rho_S, \Delta'_{GP}\right]$. Since in equilibrium, if the GP does not adopt the capital-constrained arrangement, he must choose between the second-best arrangement and the long duration arrangement, his payoff by turning away from the capital-constrained arrangement equals $\max\left[\min[\rho_S, \Delta'_{GP}], \rho_L\right]$. Thus, to prove the first half of Proposition 5, we only need to show that the change in his payoff by turning away from the capital-constrained arrangement, which is $\max[\min[\rho_S, \Delta'_{GP}], \rho_L] - \rho_C$, is increasing in τ. Note that this holds if the following three results hold: (1) $\rho_S - \rho_C$ is increasing in τ, (2) $\Delta'_{GP} - \rho_C$ is increasing in τ, and (3) $\rho_L - \rho_C$ is increasing in τ. Below we prove these three results.

Let q_{GG}, q_{GB}, and q_{BB} denote the probability of state GG occurring, of state GB occurring, and of state BB occurring, respectively. It is evident that

$$q_{GG} = \tau\lambda. \tag{52}$$

Also note that $q_{GB} = (1-\tau)\lambda + \tau'(1-\lambda)$ and $q_{BB} = (1-\tau')(1-\lambda)$. Substituting (16) into the expressions for q_{GB} and q_{BB}, we have

$$q_{GB} = 2(1-\tau)\lambda, \tag{53}$$

$$q_{BB} = 1 - 2\lambda + \tau\lambda. \tag{54}$$

It is thus clear that

$$\rho_S = q_{GG}(2R - 2I) + q_{GB}(R - I) + q_{BB}(pR - I), \tag{55}$$

$$\rho_C = (q_{GG} + q_{GB})(R - I) + q_{BB}(pR - I), \tag{56}$$

$$\rho_L = q_{GG}(2R - 2I) + q_{GB}(R - I) + q_{BB}(2 - p)(pR - I). \tag{57}$$

Thus, by (52), (53), and (54),

$$\rho_S - \rho_C = \tau\lambda(R - I), \tag{58}$$

$$\rho_L - \rho_C = \tau\lambda(R - I) - (1 - 2\lambda + \tau\lambda)(1 - p)(I - pR). \tag{59}$$

It is obvious that $\rho_S - \rho_C$ is strictly increasing in τ. Note that the first order derivative of $\rho_L - \rho_C$ with respect to τ equals $\lambda[(R - I) - (1 - p)(I - pR)]$, which must be strictly positive as $R > 2I$. Thus $\rho_L - \rho_C$ is also strictly increasing in τ. Hence, to complete the proof of the first half of Proposition 5, we only need to show that $\Delta'_{GP} - \rho_C$, is increasing in τ.

Note that

$$\Delta'_{GP} = q_{GG}s_{GP}(2R) + (q_{GB} + q_{BB}p)s_{GP}(R + I), \tag{60}$$

where q_{GG}, q_{GB}, and q_{BB} are given by (52), (53), and (54), respectively, and s_{GP} is the GP security which implements the second best without (BE) and gives the GP the maximum expected payoff. This security is presented by Lemmas 5, 6, 7, 8, and 9 under different sets of parametric assumptions.

If $R/I \leq 3-2p$, by Lemmas 5 and 8, the GP security which implements the second best without (BE) and gives the GP the maximum expected payoff has $s_{GP}(R+I) = R-I$, and $s_{GP}(2R) = 2R-2I$. Thus, by (56) and (60),

$$\frac{\partial(\Delta'_{GP} - \rho_C)}{\partial\tau} = \lambda(R - pI), \tag{61}$$

which is positive, implying that if $R/I \leq 3-2p$, $\Delta'_{GP} - \rho_C$ is strictly increasing in τ.

If $3-2p < R/I \leq 3$, by Lemmas 6 and 9, the GP security which implements the second best without (BE) and gives the GP the maximum expected payoff either has (i) $s_{GP}(R+I) = R-I$ and $s_{GP}(2R) = R-I+(3I-R)(1-p)/p$ or has (ii) $s_{GP}(R+I) = (I-pR)/(1-p)+I$ and $s_{GP}(2R) = (I-pR)/(1-p)+R$. If we denote by $\Delta^{(i)}_{GP}$ and $\Delta^{(ii)}_{GP}$ the GP's expected payoff offered by the GP security in the form of (i) and (ii) respectively, then $\Delta'_{GP} = \max\left[\Delta^{(i)}_{GP}, \Delta^{(ii)}_{GP}\right]$. It is clear that $\Delta'_{GP} - \rho_C$ is increasing in τ if both $\Delta^{(i)}_{GP} - \rho_C$ and $\Delta^{(ii)}_{GP} - \rho_C$ are increasing in τ. This is true, because

$$\frac{\partial(\Delta^{(i)}_{GP} - \rho_C)}{\partial\tau} = \lambda\left[\frac{(3I-R)(1-p)}{p} + I(1-p)\right], \tag{62}$$

which is positive as we have $3I \geq R$ in this case, and

$$\frac{\partial(\Delta^{(ii)}_{GP} - \rho_C)}{\partial\tau} = \lambda(2R - 3I + pI), \tag{63}$$

which is positive as $R/I \geq 2$.

If $R/I > 3$, by Lemmas 7 and 9, the GP security which implements the second best without (BE) and gives the GP the maximum expected payoff either has (a) $s_{GP}(R+I) = s_{GP}(2R) = 2I$ or has (b) $s_{GP}(R+I) = (I-pR)/(1-p)+I$ and $s_{GP}(2R) = (I-pR)/(1-p)+R$. Note that form (b) is the same as form (ii) just discussed above, so the GP's expected payoff offered by the GP security in the form of (b) is $\Delta^{(ii)}_{GP}$. If we denote by $\Delta^{(a)}_{GP}$ the GP's expected payoff offered by the GP security in the form of (a), then $\Delta'_{GP} = \max\left[\Delta^{(a)}_{GP}, \Delta^{(ii)}_{GP}\right]$. As we have proved that $\Delta^{(ii)}_{GP} - \rho_C$ is increasing in τ, to prove that $\Delta'_{GP} - \rho_C$ is increasing in τ, we only have to show that $\Delta^{(a)}_{GP} - \rho_C$ is increasing in τ. This is true, because

$$\frac{\partial(\Delta^{(a)}_{GP} - \rho_C)}{\partial\tau} = \lambda(1-p)(R - 2I), \tag{64}$$

which is positive as $R > 2I$.

Therefore, an increase in τ increases $\Delta'_{GP} - \rho_C$. The first half of Proposition 5 is thereby proved.

Given that an increase in τ makes the capital-constrained arrangement less favorable relative to the second-best and to the long duration arrangement, the second half of Proposition 5 follows if we can show the following: the GP never uses the capital-constrained arrangement when $\tau = 1$. This is indeed true. The reason is as follows. The GP security that has $s_{GP}(R) = 0$, $s_{GP}(R + I) = I$, and $s_{GP}(2R) = R$ can implement the second best (with (BE) omitted) and the GP's payoff offered by this security when $\tau = 1$ equals $\lambda R + (1 - \lambda)pI$. Thus, the GP's payoff by using the second-best arrangement when $\tau = 1$ must be at least $\min[\rho_S, \lambda R + (1 - \lambda)pI]$. Since $\rho_S > \rho_C$ for all τ, and also since, when $\tau = 1$, ρ_C equals $\lambda(R - I) + (1 - \lambda)(pR - I)$, which is strictly less than $\lambda R + (1 - \lambda)pI$, the GP's payoff by using the second-best arrangement must be strictly greater than his payoff by using the capital-constrained arrangement when $\tau = 1$. Therefore, the GP never uses the capital-constrained arrangement when $\tau = 1$. $\square$

Proof of Proposition 6: Let r denote the rents the GP offers to LPs by using the second-best arrangement. Note that r is not the rents asked by LPs but the rents resulting from the security design problem associated with the second-best implementation. Let ρ_2 and ρ_3 denote the second-best and the third-best surplus respectively and thereby the rents asked by LPs equal $\delta\rho_2$. The GP's payoff by using the second-best arrangement is $\rho_2 - \max[r, \delta\rho_2]$ while his payoff by using the third-best arrangement is $\rho_3 - \delta\rho_2$. Hence, the difference equals $\rho_2 - \rho_3 - \max[r - \delta\rho_2, 0]$, which is strictly increasing in δ for $\delta < r/\rho_2$ and constant in δ for $\delta \geq r/\rho_2$. Therefore, an increase in δ (weakly) increases the GP's payoff from the second-best arrangement relative to his payoff from the third-best arrangement. The first half of Proposition 6 thus follows by noting that the third-best arrangement is the GP's best choice among all the non-second-best arrangements. The second half follows by noting that, when $\delta = 1$, LPs capture all the investment surplus and they thus implement the second best by offering the GP almost nothing. $\square$

Proof of Proposition 7: As the second-best arrangement is the most efficient arrangement, financing is feasible only if the investment surplus from the second-best arrangement is nonnegative, which requires $R/I \geq \pi_0(p, \lambda)$. It is easy to check that the contract expressed by

(18) satisfies (LL), (M), (NP), and (17). Note that LPs' expected payoff is strictly decreasing in α_S. When $\alpha_S = 0$, LPs' expected payoff equals the second-best surplus expressed as (5). When $\alpha_S = 1$, LPs' expected payoff is negative. Thus, when $R/I \geq \pi_0(p, \lambda)$, implying that (5) is nonnegative, there always exists a unique α_S such that LPs just break even under the second-best arrangement. As the second-best arrangement minimizes agency conflicts and offers LPs no rents, it must minimize the GP's financing costs and will be adopted by the GP when $R/I \geq \pi_0(p, \lambda)$. $\square$

Proof of Proposition 8: We first consider the GP's payoff by using the second-best arrangement. Because of (NP), all the marginal cash flows below $2I$ must be pledged to LPs. We thus only need to consider how to pledge marginal cash flows in the cash flow regions $[2I, R+I]$ and $[R+I, 2R]$. It is evident that to satisfy condition (17), it is best to pledge to the GP the marginal cash flows in the region $[2I, R+I]$ first and then the marginal cash flows in the region $[R+I, 2R]$. Without considering the LPs' break-even condition, among all the securities that satisfy (LL), (M), (NP), and (17), the GP's expected payoff is maximized by the security that satisfies $s_{GP}(R+I) = R - I$ and $s_{GP}(2R) = \frac{R-I}{p}$. This security offers the GP the expected payoff equal to

$$\left[\frac{\lambda^2}{p} + 2\lambda(1-\lambda) + (1-\lambda)^2 p\right](R - I). \tag{65}$$

Thus, if $R/I \geq \pi_0(p, \lambda)$, by using the second-best arrangement, the GP's expected payoff equals the minimum of (5) and (65).

Next, we show that the GP always prefers the second-best arrangement to the capital-constrained arrangement. This follows because the GP's expected payoff by using the capital-constrained arrangement, expressed by (10), is strictly less than both (5) and (65). Note that (10) is strictly less than (65) so long as

$$\frac{R}{I} > \frac{\frac{\lambda^2}{p} + 2\lambda - 2\lambda^2 + (1-\lambda)^2 p - 1}{\frac{\lambda^2}{p} - \lambda^2}. \tag{66}$$

The righthand side of (66) is strictly less than 1 while the lefthand side of (66) is strictly greater than 1 by assumption. Thus (66) must hold and (10) must be strictly less than (65).

Then we examine the GP's payoff by using the long duration arrangement. Under the long duration arrangement, the GP (weakly) prefers making investment decisions sequentially rather than simultaneously. As $s_{GP}(R) = 0$, in state BB, if his earlier investment fails, the GP has no incentive to invest in the other bad project. The second-best is implementable under the long duration arrangement if the GP security satisfies (IC_{GB}) while (IC_{GB}) becomes (17) when $s_{GP}(R) = 0$. Hence the security design problem associated with implementing the second-best under the long duration arrangement is exactly the same as that under the short duration arrangement. Therefore, implementing the second-best under the long duration arrangement gives the GP the same payoff as that under the short duration arrangement. This implies that the only possible situation for the GP to favour the long duration arrangement corresponds to the long duration arrangement with (17) omitted. Without (17), marginal cash flows above $2I$ can be flexibly pledged to the GP and thus LPs earn zero rents. The GP's investment behavior is as follows: he invests in all projects except in state BB, if his earlier investment fails, he abandons the other bad project. The GP's expected payoff from this investment behavior equals the associated investment surplus, expressed as

$$\lambda^2(2R - 2I) + 2\lambda(1-\lambda)(R + pR - 2I) + (1-\lambda)^2(1+p)(pR - I). \tag{67}$$

Note that the long duration arrangement is preferred to the second-best arrangement by the GP if and only if expression (67) is greater than expression (65), which is equivalent to $R/I > \pi'_{SL}(p,\lambda)$, where $\pi'_{SL}(p,\lambda)$ is given by the following

$$\pi'_{SL}(p,\lambda) = \frac{1 + \lambda^2 - \frac{\lambda^2}{p}}{(1-\lambda)^2 p^2 + 2\lambda p - 2\lambda^2 p + 2\lambda^2 - \frac{\lambda^2}{p}}. \tag{68}$$

In addition, if condition (19) is satisfied, $\pi'_{SL}(p,\lambda) \geq 1/p$, in which case the second-best arrangement is preferred to the long duration arrangement for all $R/I < 1/p$. If condition (19) is violated, $1 < \pi'_{SL}(p,\lambda) < 1/p$, in which case the second-best arrangement is preferred to the long duration arrangement for all $R/I \leq \pi'_{SL}(p,\lambda)$ while the long duration arrangement is preferred to the second-best for all $\pi'_{SL}(p,\lambda) < R/I < 1/p$.

Finally, note that under the stand-alone arrangement, the GP never abandons any bad project, so the GP always prefers the long duration arrangement to the stand-alone arrangement. Moreover, under the non-second-best arrangement, condition (17) is violated, which implies that both (IC_{BB}) and (IC_{GB}) are violated. Thus, the GP never abandons any bad project under the non-second-best arrangement. Hence the GP always prefers the long duration arrangement to the second-best arrangement. □

Appendix B: An Example Related to Section 4.1

Here we provide an example to show that, all else being equal, an increase in project quality correlation can sometimes induce the GP to switch from the second-best arrangement to the long duration arrangement.

Suppose $\lambda = 0.1$, $p = 0.2$, and $R/I = 4.8$. First consider the situation where the two projects have independent quality, i.e., where $\tau = \lambda$. In this case, both (34) and (51) hold while (50) does not hold. Thus, according to the proof of Theorem 1, the GP prefers the second-best arrangement if $R/I \leq \pi_{SL}(p, \lambda)$, where $\pi_{SL}(p, \lambda)$ is expressed by (49). Notably, under the current assumptions, $R/I < \pi_{SL}(p, \lambda)$, so the GP prefers the second-best arrangement. Note also that the second-best arrangement is feasible if $R/I \geq \pi_0(p, \lambda)$, where $\pi_0(p, \lambda)$ is expressed in (22). This condition holds under the current parametric assumptions. Hence, under the current assumptions, the GP raises funds by using the second-best arrangement.

Fixing the above parametric assumptions, now consider the situation where the two projects have perfectly correlated quality, i.e., where $\tau = 1$. By Lemmas 7 and 9, when $R/I > 3$, to implement the second best without taking into account investors' break-even condition (BE), the GP security which maximizes the GP's payoff must either have (a) $s_{GP}(R + I) = s_{GP}(2R) = 2I$ or have (b) $s_{GP}(R + I) = (I - pR)/(1 - p) + I$, and $s_{GP}(2R) = (I - pR)/(1 - p) + R$. The GP's payoff if the second best is implemented has the following expression:

$$\lambda s_{GP}(2R) + (1 - \lambda)p s_{GP}(R + I). \tag{69}$$

It is easy to see that, under the current parametric assumptions, the GP's payoff equals $0.56I$ if the GP security takes the form (a) while equals $0.674I$ if it takes the form (b). Thus, if we take into account the investors' break-even condition (BE), the GP's payoff by using the second-best arrangement must be the minimum of $0.674I$ and the second-best surplus. Note that when the two projects have perfectly correlated quality, if the GP uses the long duration arrangement, he captures all the investment surplus in the following expression:

$$\lambda(2R - 2I) + (1 - \lambda)(2 - p)(pR - I), \tag{70}$$

which, under the current parametric assumptions, equals $0.6952I$ and is thus greater than the GP's payoff by using the second-best arrangement. Hence, the GP raises funds by using the long duration arrangement when the two projects have perfectly correlated quality.

References

[1] Axelson, Ulf, Per Strömberg, and Michael S. Weisbach, 2009, Why are buyouts levered? The financial structure of private equity funds, *The Journal of Finance* 64, 1549-82.

[2] Bolton, Patrick, and David S. Scharfstein, 1990, A theory of predation based on agency problems in financial contracting, *The American Economic Review* 80, 93-106.

[3] Cumming, Douglas J., and Sofia A. Johan, 2009, *Venture Capital and Private Equity Contracting: An International Perspective* (Elsevier Science Adademic Press).

[4] Da Rin, Marco, Thomas Hellmann, and Manju Puri, 2011, A survey of venture capital research, Working paper.

[5] Dass, Nishant, Hung-Chia Hsu, Vikram Nanda, and Qinghai Wang, 2012, Churning in venture capital investments, Working paper.

[6] DeMarzo, Peter M., and Michael J. Fishman, 2007, Optimal long-term financial contracting, *The Review of Financial Studies* 5, 2079-128.

[7] Diamond, Douglas, 1984, Financial intermediation and delegated monitoring, *The Review of Economic Studies* 51, 393-414.

[8] Fulghieri, Paolo, and Merih Sevilir, 2009, Size and focus of a venture capitalist's portfolio, *The Review of Financial Studies* 22, 4643-80.

[9] Gompers, Paul, and Josh Lerner, 1996, The use of covenants: An empirical analysis of venture partnership agreements, *The Journal of Law and Economics* 39, 463-98.

[10] Gompers, Paul, and Josh Lerner, 1999, An analysis of compensation in the U.S. venture capital partnership, *Journal of Financial Economics* 51, 3-44.

[11] Harris, Lee, 2010, A critical theory of private equity, *Delaware Journal of Corporate Law* 35, 259-93.

[12] Harris, Milton, and Artur Raviv, 1989, The design of securities, *Journal of Financial Economics* 24, 255-287.

[13] Inderst, Roman, Holger Mueller, and Felix C. Münnich, 2007, Financing a portfolio of projects, *The Review of Financial Studies* 20, 1289-325.

[14] Kandel, Eugene, Dima Leshchinskii, and Harry Yuklea, 2011, VC funds: Aging brings myopia, *Journal of Financial and Quantitative Analysis* 46, 431-57.

[15] Kaplan, Steven N., and Antoinette Schoar, 2005, Private equity performance: Returns, persistence and capital flows, *Journal of Finance* 60, 1791-824.

[16] Laux, Christian, 2001, Limited-liability and incentive contracting with multiple projects, *The Rand Journal of Economics* 32, 514-26.

[17] Litvak, Kate, 2004, Governance through exit: Default penalities and walkaway options in venture capital partnership agreements, *Willamette Law Review* 40, 771-812.

[18] Nachman, David C., and Thomas H. Noe, 1994, Optimal design of securities under asymmetric information, *The Review of Financial Studies* 7, 1-44.

[19] Noe, Thomas H., and Jun Wang, 2004, Fooling all of the people some of the time: A theory of endogenous sequencing in confidential negotiations, *The Review of Economic Studies* 71, 855-81.

[20] Osborne, Martin J., and Ariel Rubinstein, 1990, *Bargaining and markets* (Academic Press).

[21] Phalippou, Ludovic, 2007, Investing in private equity funds: A survey, Working paper.

[22] Sahlman, William, 1990, The structure and governnance of venture capital organizations, *Journal of Financial Economics* 27, 473-521.

[23] Stein, Jeremy, 1997, Internal capital markets and the competition for corporate resources, *The Journal of Finance* 52, 111-33.

[24] Tirole, Jean, 2006, *The theory of corporate finance* (Princeton University Press).